Simulation of Wave front propagation with ZEMAX

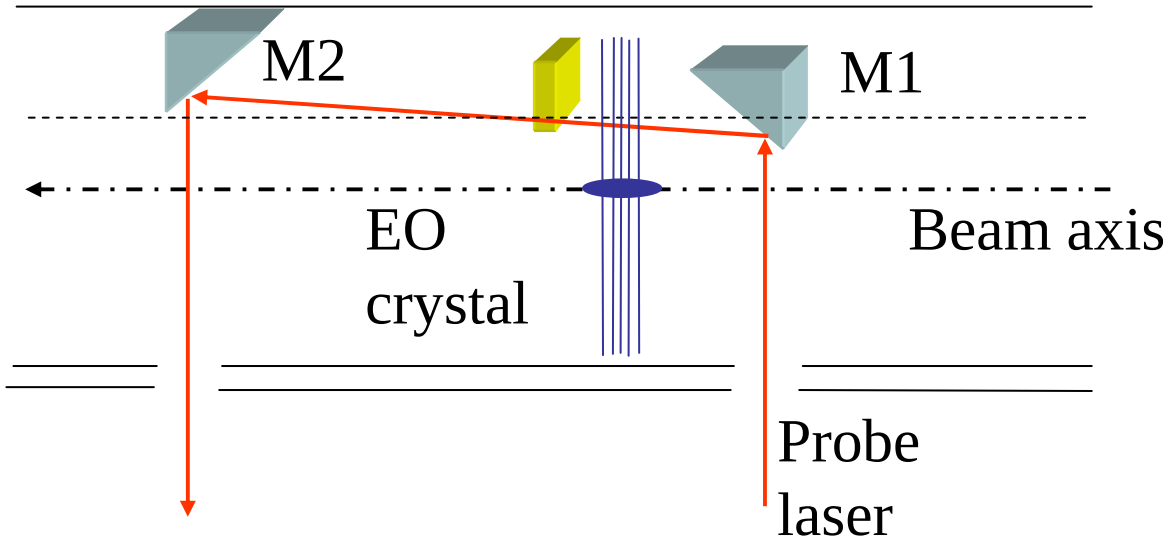
Sara Casalbuoni and Rasmus Ischebeck
DESY

Overview

- Motivation
  - FEL optics (S. Düsterer) λ = 6 nm
  - TEO optics λ = 800 nm
  - THz radiation λ = 1 mm
- Tool: ZEMAX
  - Optics
  - Usage
- First results
  - TEO optics
  - Free Propagation in Near Field
  - CTR on paraboloid mirror
- Future plans

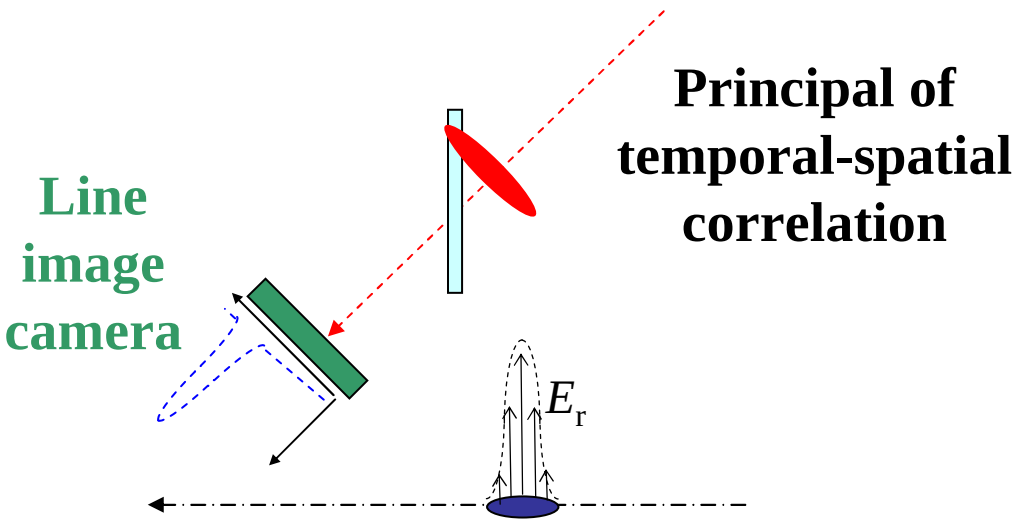
The TEO experiment

- *Pockels effect: E_r of e^--bunch rotates laser polarization*

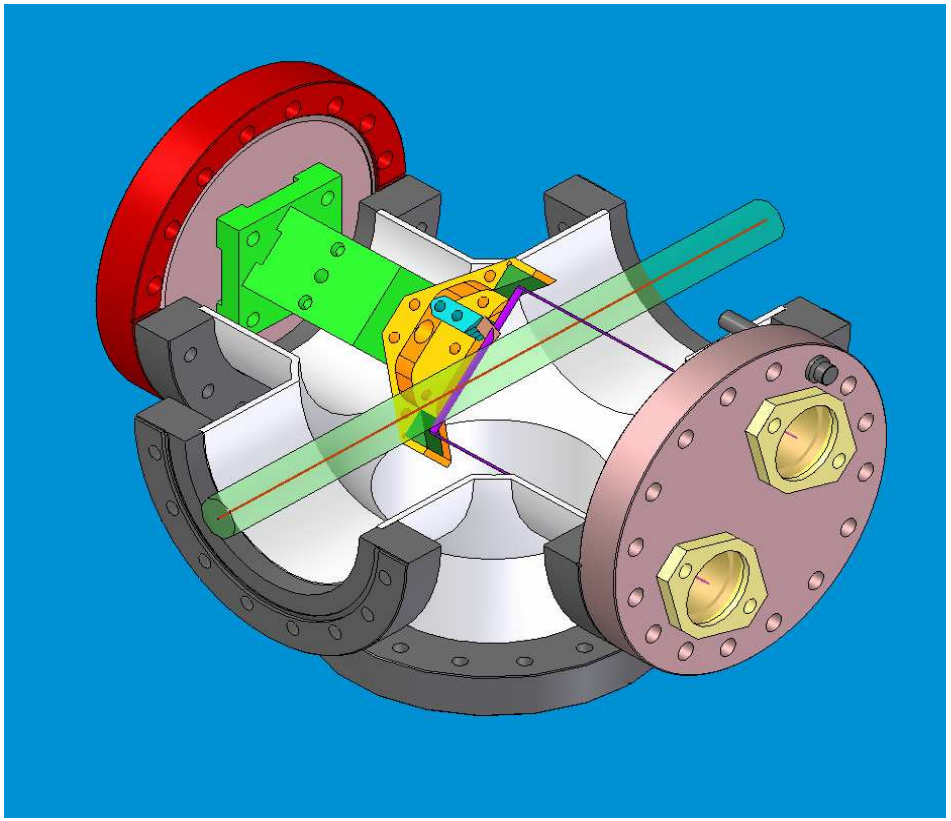


Experiment prepared by
Univ. Michigan, D. Reis, A .Cavalieri
SLAC, with contributions from DESY

- EO vacuum chamber



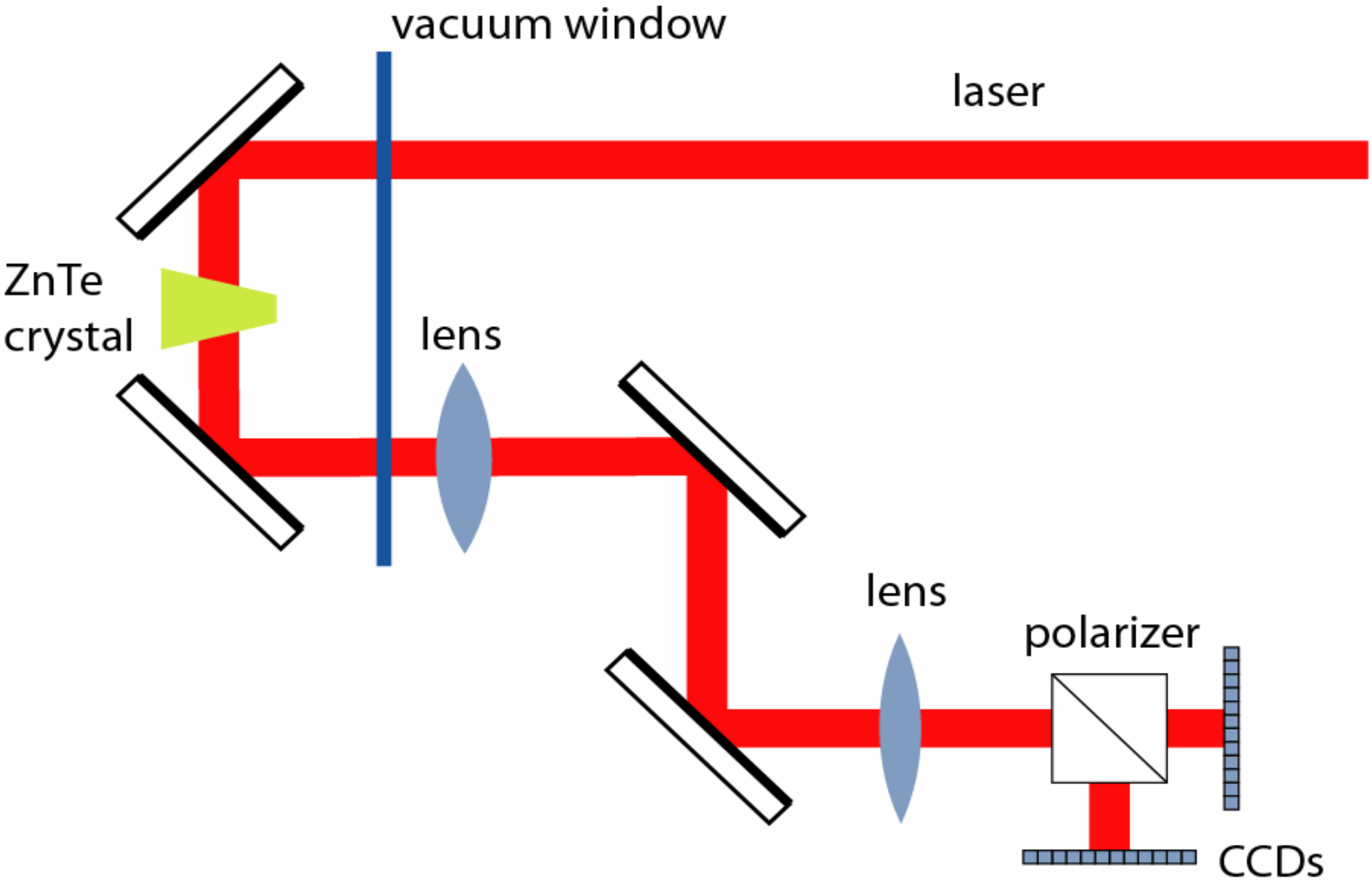
Principal of temporal-spatial correlation



Present resolution limit ~ 100 fs
(limited by laser in tunnel)

Motivation

Optics for the TEO experiment



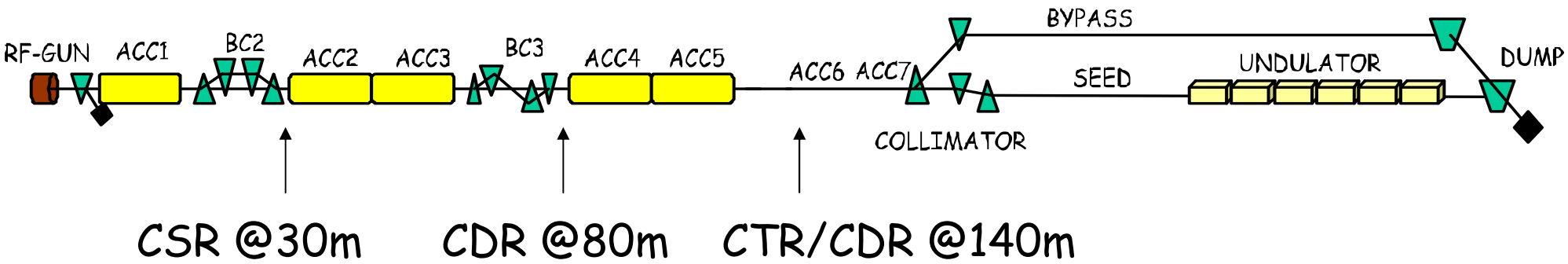
additionally, some planes are rotated by 45°

Motivation

- Beam diagnostics with coherent THz spectroscopy:
- Coherent Transition Radiation (CTR)
- Coherent Diffraction Radiation (CDR)
- Coherent Synchrotron Radiation (CSR)
- Design of transfer line for THz coherent radiation from source to interferometer
- Calculate the transfer function

Motivation

TTF2 scheme with placing of the interferometers



Transfer function

Coherent radiation contains bunch length information in the form factor

$$I_{tot}(\omega) = I(\omega)\,[N + N\,(N-1)\,|\,f(\omega)\,|^2]$$

At λ>bunch length => Coherent Emission

$$f(\omega) = \int_{-\infty}^{\infty} \rho(z)\ \exp(i\omega z/c)\ dz = \int_{-\infty}^{\infty} c\rho(ct)\ \exp(i\omega t)\ dt$$

Normalized longitudinal charge distribution

Fourier transform of the measured intensity

$$I_{meas}(\omega) = T(\omega)\,I_{tot}(\omega)$$

T(ω) can be obtained by :

- Calibrating the system with a known source
- Calculate it by ZEMAX

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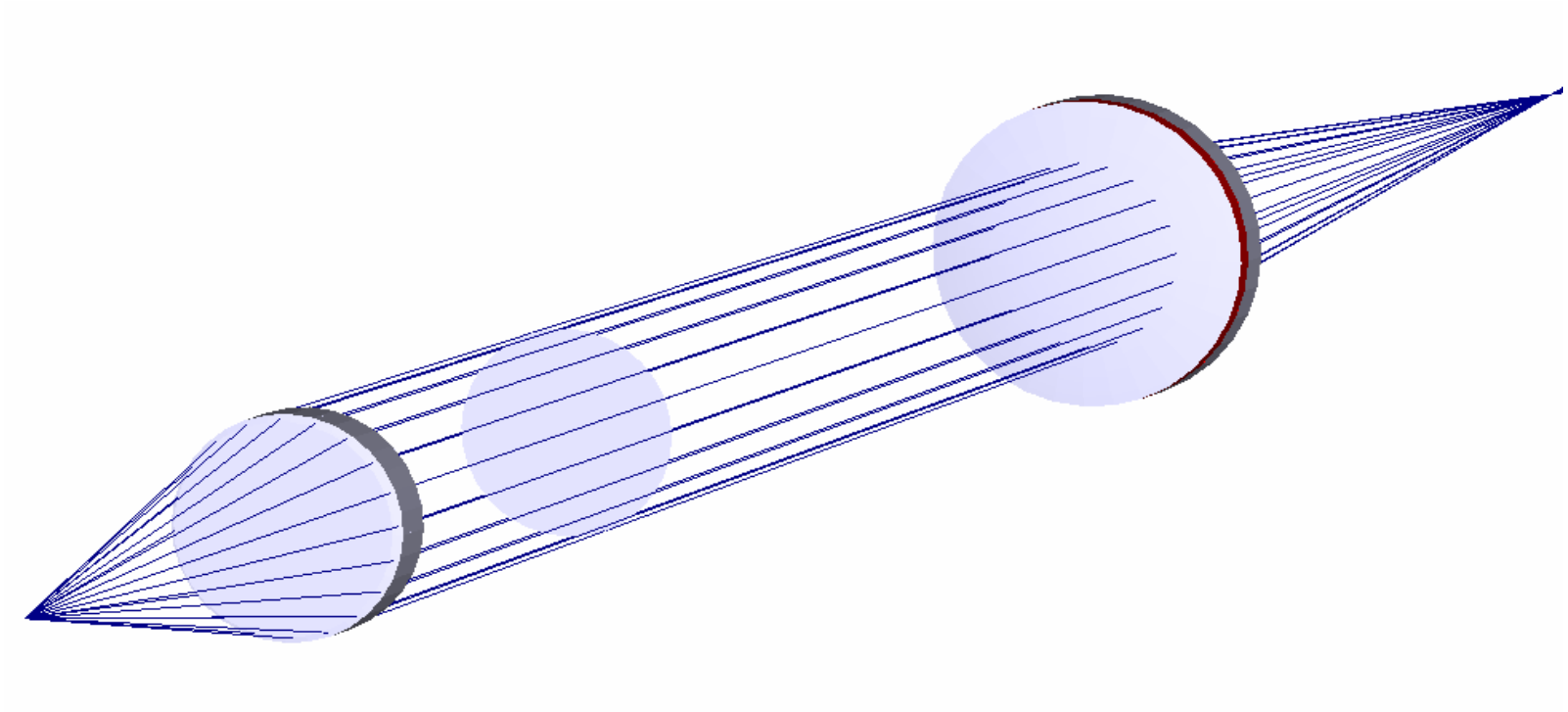
Optics

- Commercial optics
- Perfect lens optics
- Ray optics
- The Slowly Varying Amplitude approximation
- Gaussian optics
  - Near field and far field
  - Notion of the Rayleigh length and the Fresnel number
- Propagation of a wave front
  - Far field (Fraunhofer diffraction propagation)
  - Near field (Fresnel diffraction propagation)
    - called “far field” in the ZEMAX manual
  - Very near field (angular spectrum propagation)
    - called “near field” in the ZEMAX manual

Ray Optics (Geometrical Optics)

- In vacuum, light propagates on a straight line
- Refraction at the surfaces according to Snell's Law
- No diffraction



Geometrical Optics

- ZEMAX takes into account
  - “thick” lenses
  - spherical aberration at the surfaces of the lens
  - wavelength-dependent index of refraction (chromatic aberration)
- and calculates
  - beam positions and angles
  - spherical aberrations, coma, chromatic aberrations
  - contribution of the various aberrations to the resolution
  - optimization of parameters
  - etc…

The Slowly Varying Amplitude (SVA) approximation

Plan:

- Assume an electromagnetic field of the form

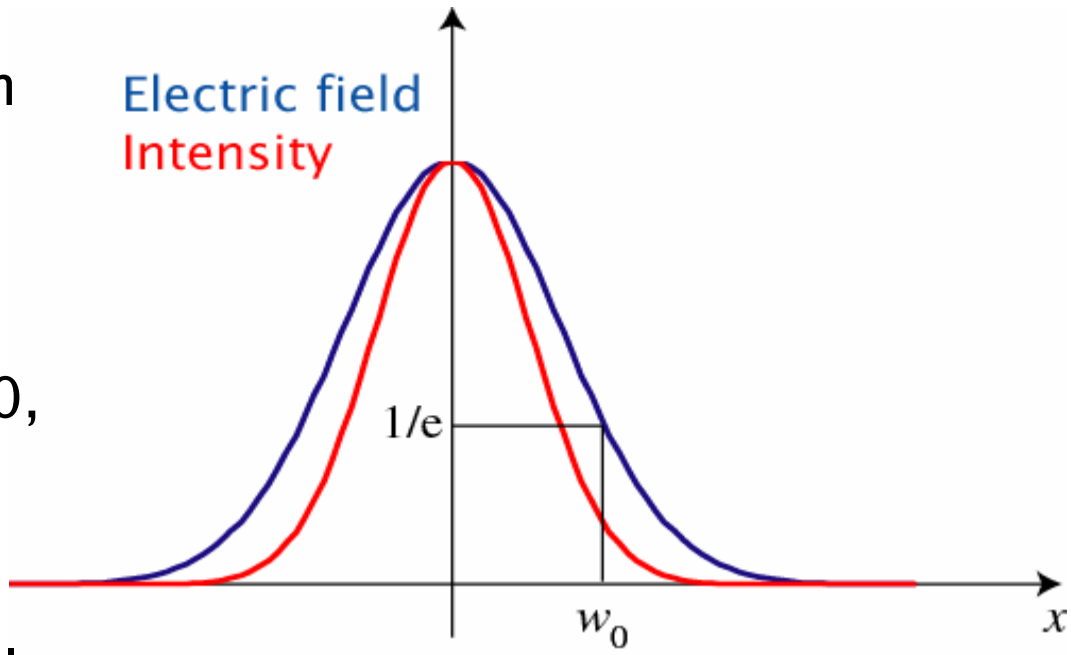
$$\vec{E}(\vec{r},t) = \mathrm{Re}\left[\tilde{E}(\vec{r},t)\exp\left(i(\omega t - \vec{k}\cdot\vec{r})\right)\right]\vec{u}_x$$

- Amplitude $\tilde{E}(\vec{r},t)$ is assumed to vary slowly
  - slowly as compared to the wavelength
  - AND slowly as compared to the grid spacing
- Derive the development (propagation) of a wave front from Maxwell's Equations
- Find clever ways to solve the propagation numerically
  - time-efficient
  - minimize numerical errors

Gaussian Optics

- Assume an amplitude of the form

$$\tilde{E}(z=0) = E_0 \cdot e^{-(x^2+y^2)/w_0^2}$$



- Assume a constant phase for z=0, corresponding to an infinite radius of curvature
- Often found in laser optics
- By solution of the Fresnel integral, one can show that the amplitude distribution will remain Gaussian when the beam propagates
- The behavior is governed by the Rayleigh length, defined as:

$$z_R = \frac{\pi w_0^2}{\lambda}$$

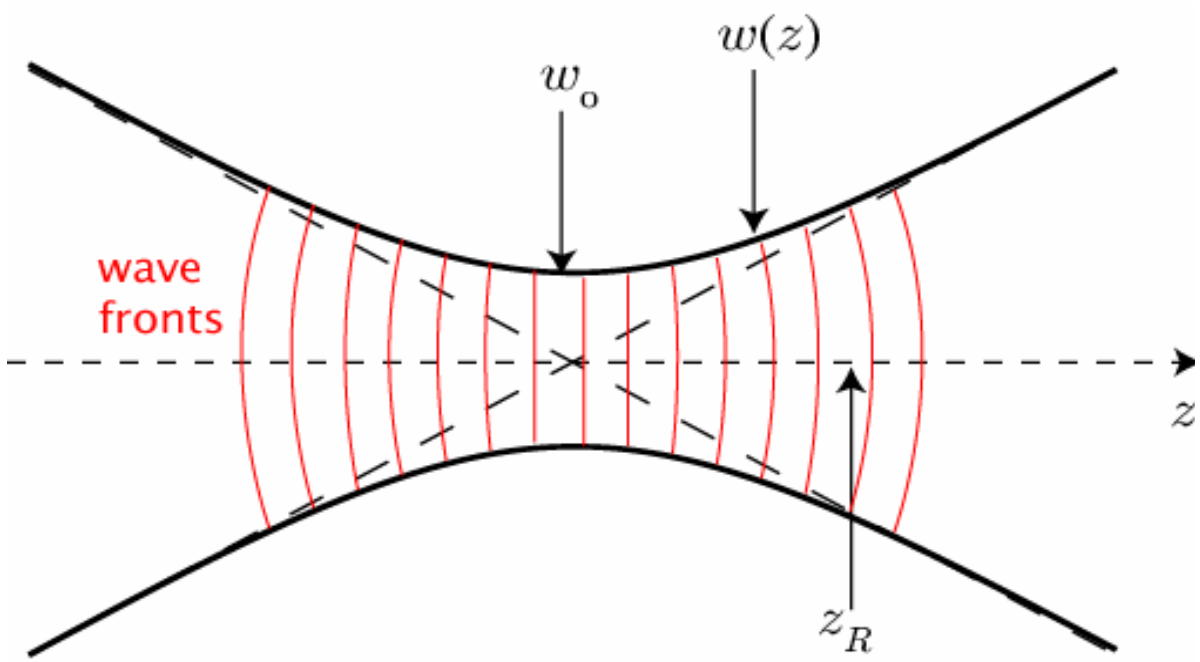
Gaussian Optics

- The wave front will acquire a curvature for $z>0$:

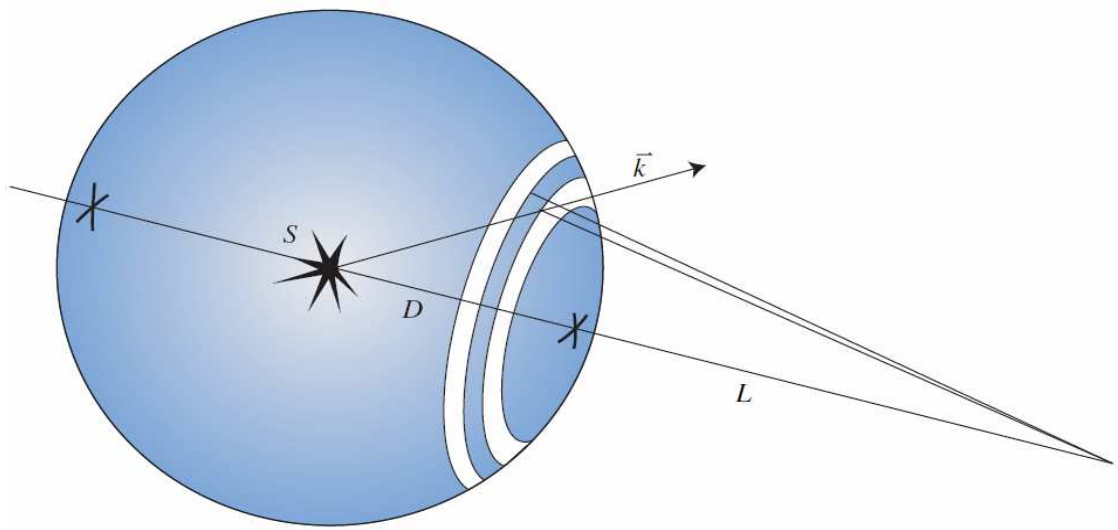
$$R(z) = z\left[1+\left(\frac{z_R}{z}\right)^2\right]$$

- The width of the beam will evolve as:

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$



Near Field and Far Field



Fresnel number

- For a beam that has a radius R, propagated by a distance L

$$N_F = \frac{2}{\lambda}\left(\sqrt{z^2 + R^2} - z\right) \approx \frac{r^2}{\lambda z}$$

- equal to the number of Fresnel rings on the source plane that can be seen from any point on the detection plane
- is a position of both the source and the detection plane

- Far field: $N_F = 0$
- Near field: $0 < N_F < 1$
- Very near field: $N_F > 1$

Propagation of an Arbitrary Wave Front

- Assume a wave with an arbitrary amplitude given numerically on a grid

$$\tilde{E}(x,y) = \begin{bmatrix} \ddots & & \\ & \ddots & \\ & & \ddots \end{bmatrix}$$

- Propagation along the z axis can be calculated in three domains:
  - Far field (Fraunhofer diffraction propagation)
  - Near field (Fresnel diffraction propagation)
    - $\rightsquigarrow$ Peter's and Bernhard's talk
  - Very near field (angular spectrum propagation)

Fraunhofer Diffraction

- Propagation from the source plane (index 0) to the observation plane (index 1)
- The electric field amplitude on the observation plane can be calculated by a superposition of the wavelets originating from all points in the source plane

$$\tilde{E}(x_1, y_1, z) = \int \tilde{E}(x_0, y_0, 0)\, \frac{e^{ik\ell}}{i\lambda\ell}\, K(\vartheta) \mathrm{d}x_0\, \mathrm{d}y_0$$

- In the far field, one may make the following approximations:

$$\tilde{E}(x_1, y_1, z) = \int \tilde{E}(x_0, y_0, 0)\, e^{ik\left(\frac{-x_1 x_0 - y_1 y_0}{L}\right)} \mathrm{d}x_0\, \mathrm{d}y_0$$

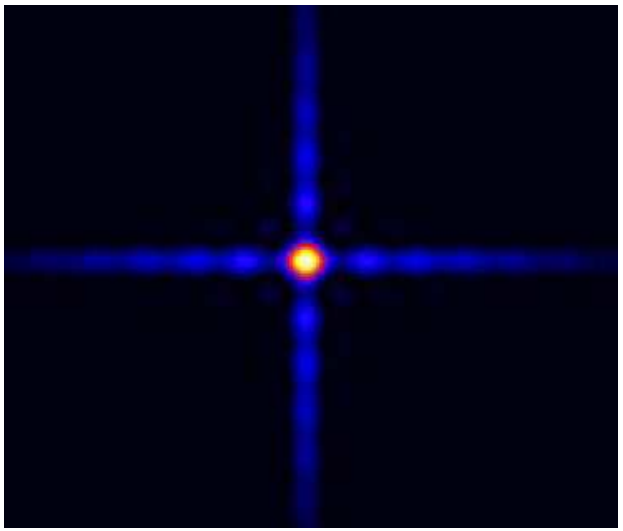
Fresnel Diffraction

- If the far field approximation cannot be made, one has to use the second order expansion:

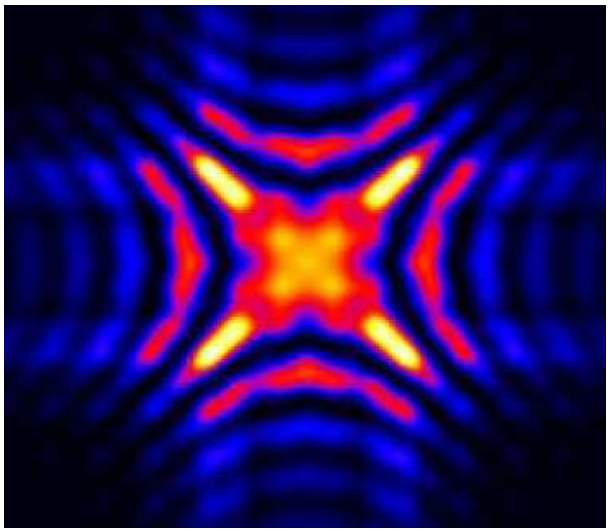
$$\tilde{E}(x_1, y_1) = \frac{1}{i\lambda L} \int \tilde{E}(x_0, y_0)\, e^{ik\left(\frac{-2x_1x_0 - 2y_1y_0 + x_0^2 + y_0^2}{2L}\right)} \mathrm{d}x_0\, \mathrm{d}y_0$$

- The near field effects can be essential:

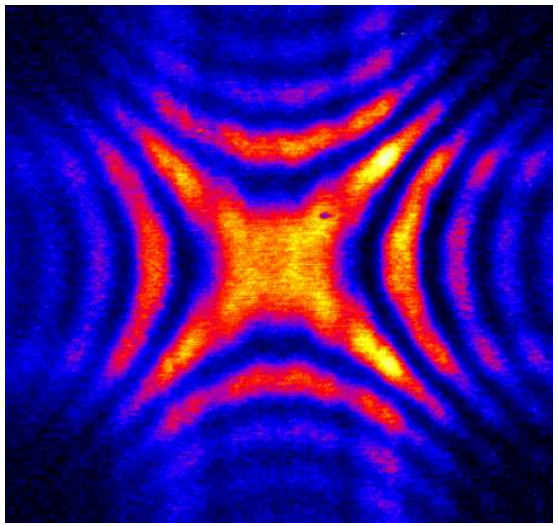
far field simulation



near field simulation



measurement



Numerical Considerations

- The integrals are represented as sums.
- The number of terms in the sum is very large, and time-intensive functions have to be computed
- If the separation of the grid points is too large, the transition is not smooth

  ⇒ Aliasing occurs

  ⇒ Increase the number of grid points

  - This problem is especially serious for large Fresnel numbers, as the phase changes rapidly in the outer Fresnel zones

Angular Spectrum Propagation

- Assume an electric field amplitude on a grid $\tilde{E}(x, y)$
- Let the Fourier transform of the field be

$$\tilde{G}(\xi, \eta) = \mathcal{F}\left\{\tilde{E}(x, y)\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{E}(x, y) \quad e^{-2\pi i(x\xi + y\eta)} \quad \mathrm{d}x\,\mathrm{d}y$$

- The inverse Fourier transform is

$$\tilde{F}(x, y) = \mathcal{F}^{-1}\left\{\tilde{G}(\xi, \eta)\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}(\xi, \eta) \quad \underbrace{e^{2\pi i(x\xi + y\eta)}} \quad \mathrm{d}\xi\,\mathrm{d}\eta$$

plane waves propagating in the direction

$$\left(\xi\lambda, \quad \eta\lambda, \quad \sqrt{1 - (\xi\lambda)^2 - (\eta\lambda)^2}\right)$$

Angular Spectrum Propagation

- The wave can be regarded as a superposition of plane waves, the field $\tilde{G}(\xi, \eta)$ is the angular spectrum.

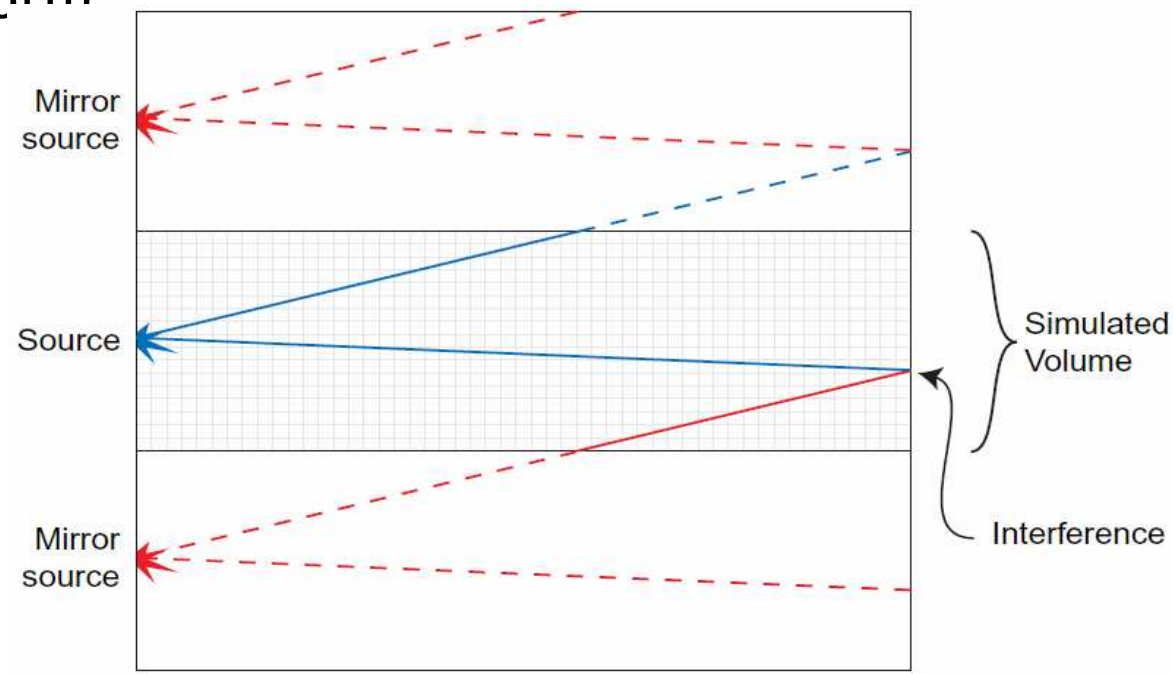
- Using the approximation that the wave propagates at a small angle to the z axis,
$$e^{2\pi i(x\xi + y\eta)} \approx e^{2\pi i z/\lambda} \cdot e^{-\pi i(\xi^2 + \eta^2)}$$

- Thus, the angular spectrum evolves as
$$\tilde{G}(\xi, \eta, z) = \tilde{G}(\xi, \eta, 0) \cdot e^{-\pi i \lambda(\xi^2 + \eta^2) z}$$

- Solving the Maxwell equations is reduced to a simple multiplication.

Limitations of the Angular Spectrum Propagator

- The size of the grid on which the field is given remains constant as the beam evolves.
  - ⇒ If the beam expands a lot during the propagation, it exceeds the grid
  - ⇒ The angular spectrum propagator is suitable for cases where the beam diameter does not change significantly, i.e. near the focus
- The use of the Fourier transform implicitly assumes a periodic source.
  - ⇒ There is an interference with rays that come from the mirror sources:
  - ⇒ One has to make the grid large enough (⇒ high demands on processor & memory)



Usage of the Algorithms in ZEMAX

Algorithm	Usage (examples)
Ray tracing	Geometrical optics: find the position of the focus
Fraunhofer diffraction	Calculation of the Point Spread Function in the focus
Fresnel diffraction	Physical Optics Propagation for small Fresnel numbers
Angular spectrum propagation	Physical Optics Propagation for large Fresnel numbers

Decision on the Fresnel numbers is based on a Gaussian pilot beam

Tool:ZEMAX

- What is ZEMAX?

  `ZEMAX is a program which can model, analyze and assist in the design of optical systems, but is NO substitute for good engineering practices´

- What do we do with ZEMAX?

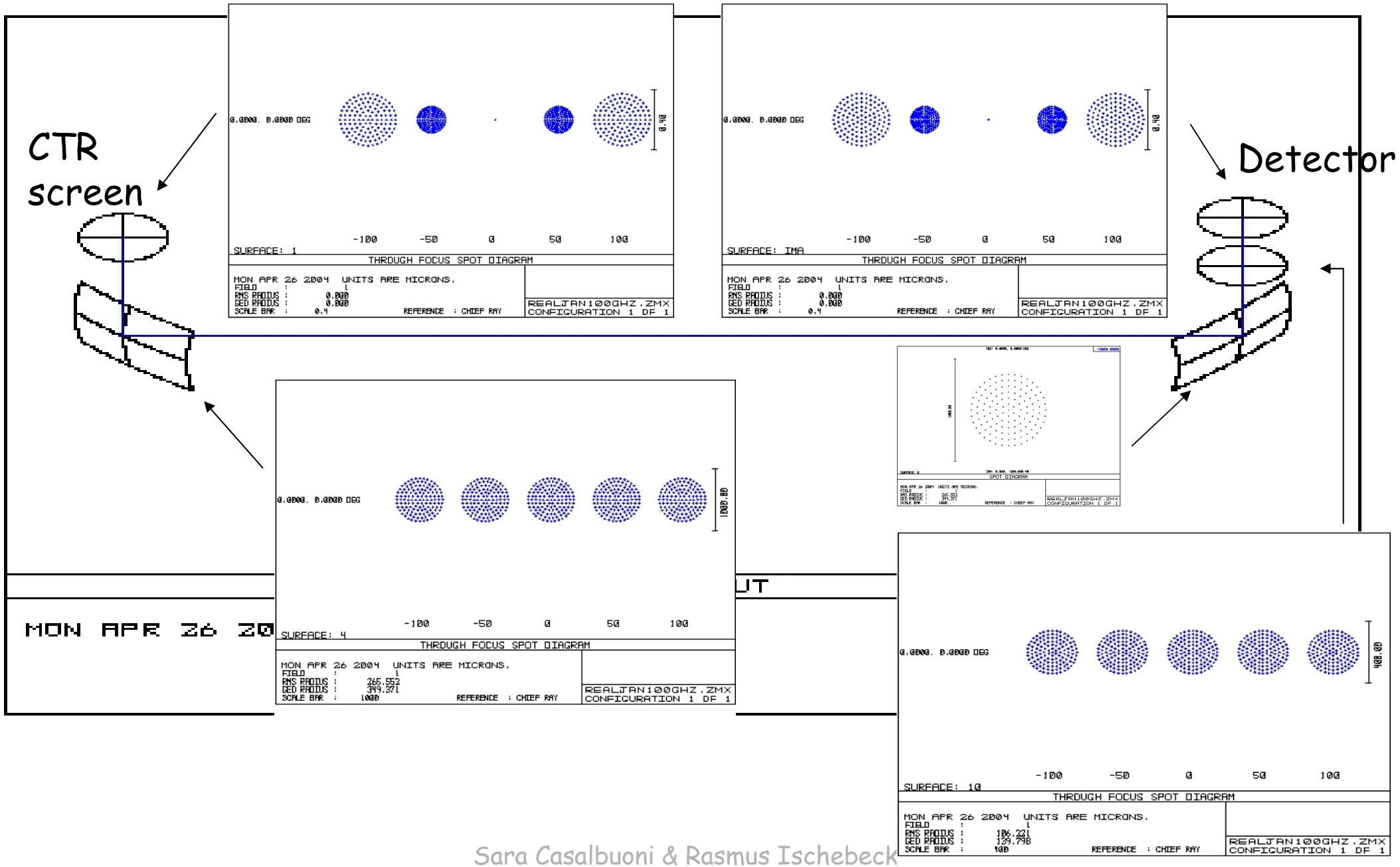
- Plug in an optical design (lenses, THz:mirrors)

- Ray tracing as first check (focal point @right positions)

- **PHYSICAL OPTICS PROPAGATION**

Tool: ZEMAX

Transferline for the Martin-Pupplet interferometer @TTF1



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PHYSICAL OPTICS PROPAGATION:Usage

- ZEMAX input:
  Fourier Transform of the Electric field on the first surface in the 2 polarization directions $\tilde{E}_x$, $\tilde{E}_y$

- Example for CTR:
  Fourier Transform of the radial Electric field of a uniform bunch charge distribution moving with velocity v in straight line uniform motion from M. Geitz PhD Thesis

$r = \text{radial coordinate} \qquad R = \text{beam radius} \qquad b = \text{pipe radius} \qquad k = 2\pi/\lambda = \omega/c$

$$\tilde{E}(k,r) = r \cdot I_1(kr/\gamma) \cdot K_1(kR/\gamma) + R \cdot I_1(kr/\gamma) \cdot I_1(kR/\gamma) \frac{K_0(kb/\gamma)}{I_0(kb/\gamma)}; \quad r < R$$

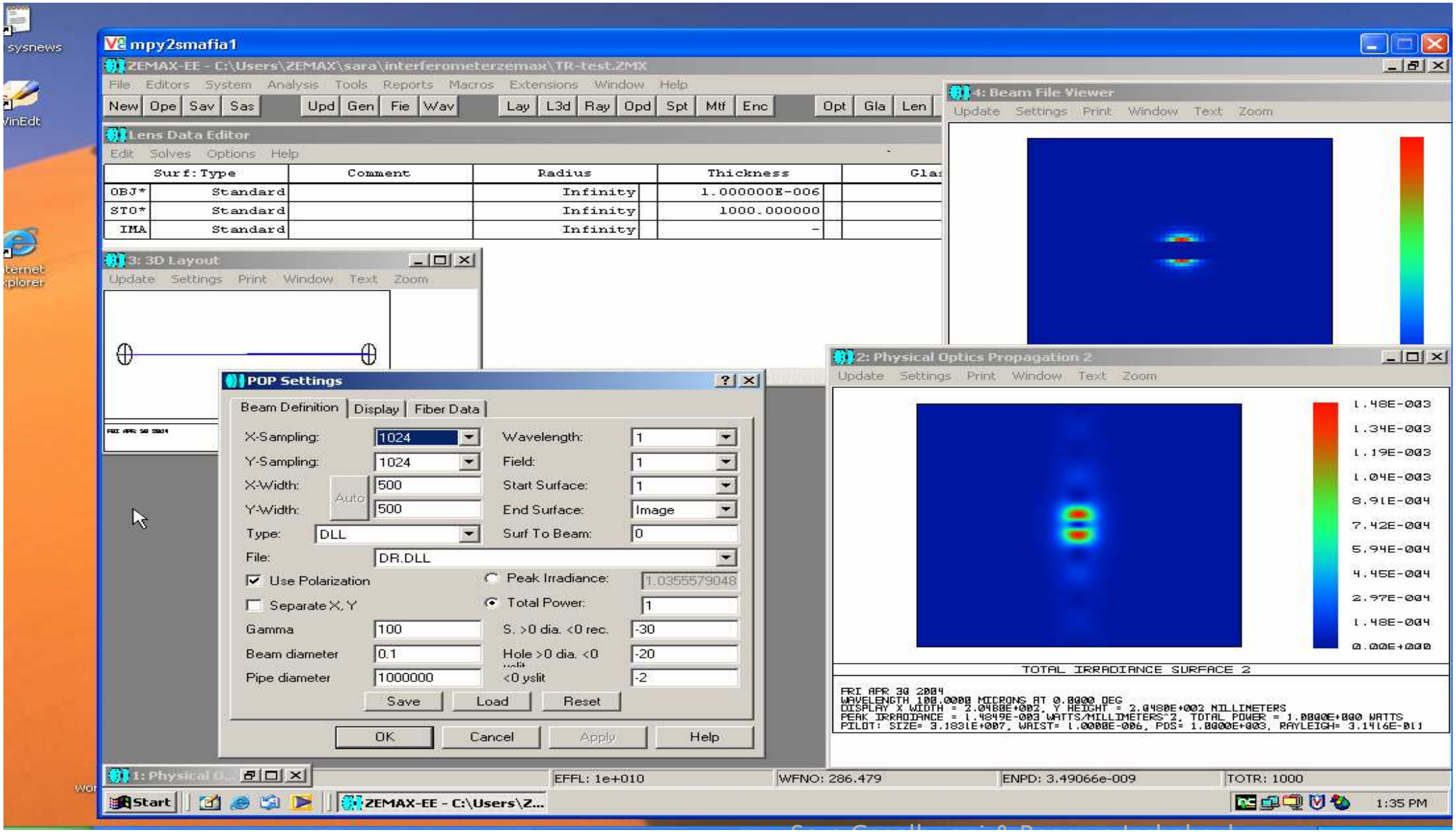
$$\tilde{E}(k,r) = R \cdot I_1(kR/\gamma) \cdot K_1(kr/\gamma) + R \cdot I_1(kr/\gamma) \cdot I_1(kR/\gamma) \frac{K_0(kb/\gamma)}{I_0(kb/\gamma)}; \quad r > R$$

$$\varphi = \text{atan}(y/x)$$

$$\tilde{E}_x = \tilde{E}_r \cdot \cos\varphi \qquad \tilde{E}_y = \tilde{E}_r \cdot \sin\varphi$$

PHYSICAL OPTICS PROPAGATION:Usage

- Source field (J. Menzel implemented .DLL for CTR and CDR)



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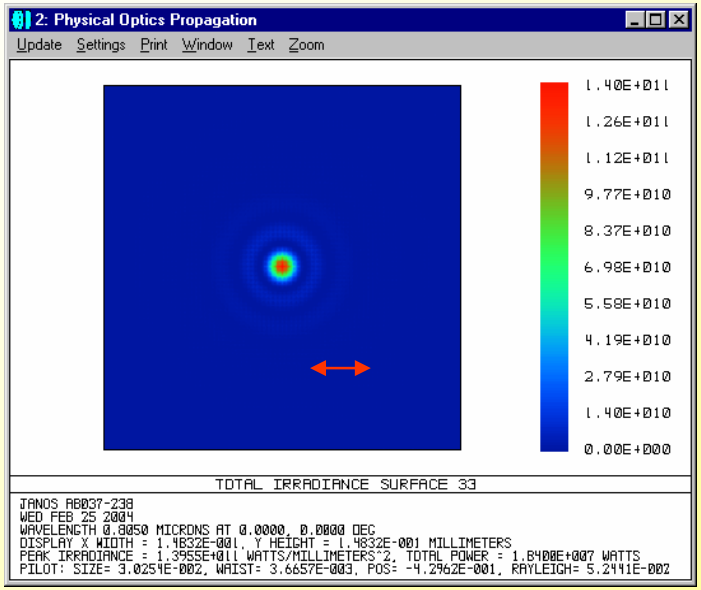
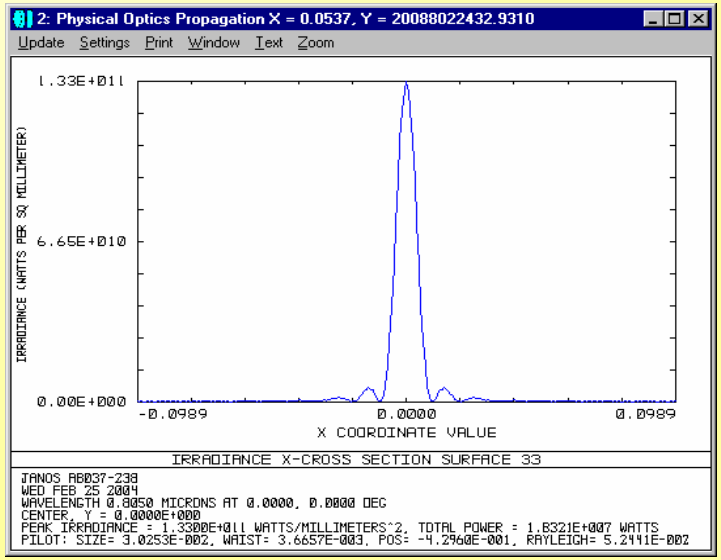
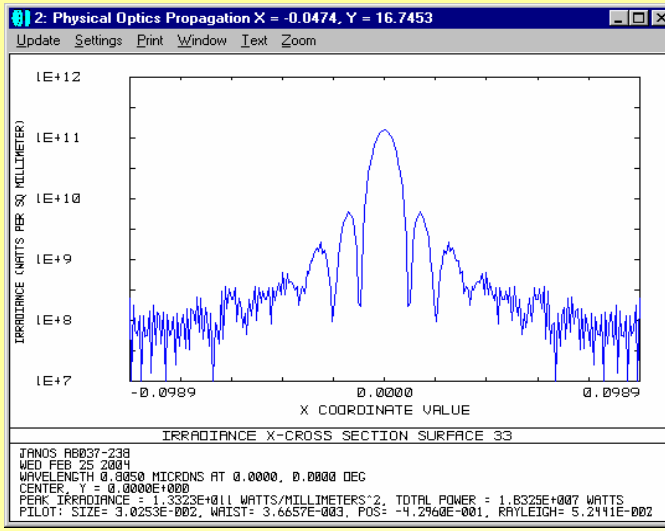
the real focus – using real lens telescopes



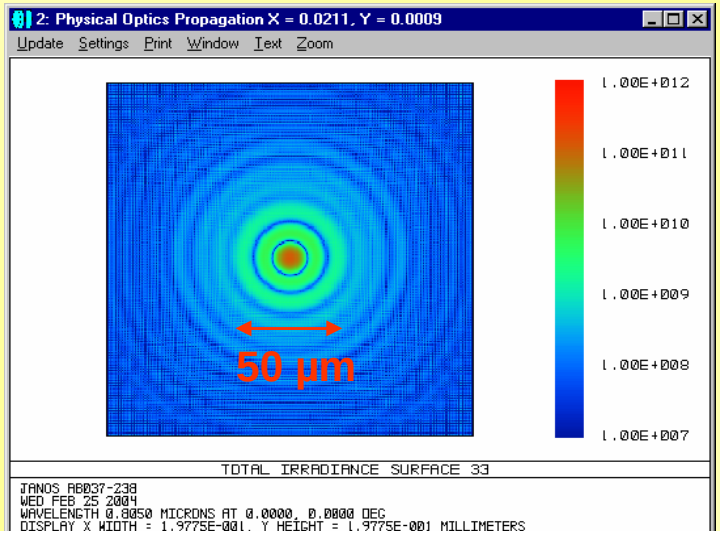
1-D cut - 9µm focus



1-D cut - logarithmic

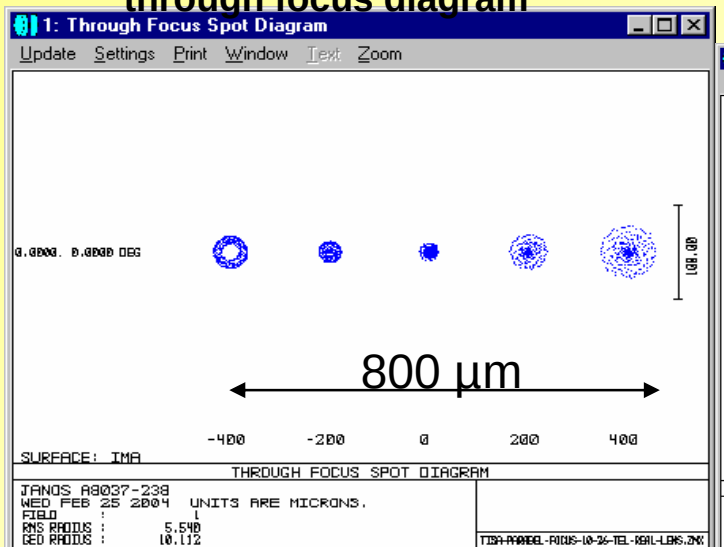


logarithmic focus image



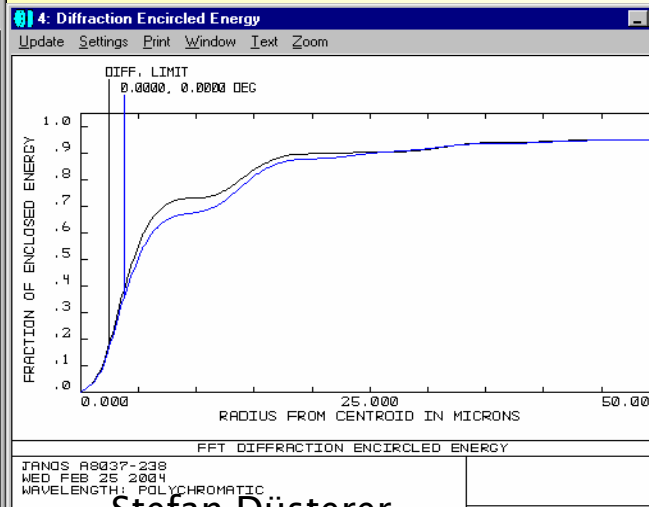
geometrical focus variation

through focus diagram



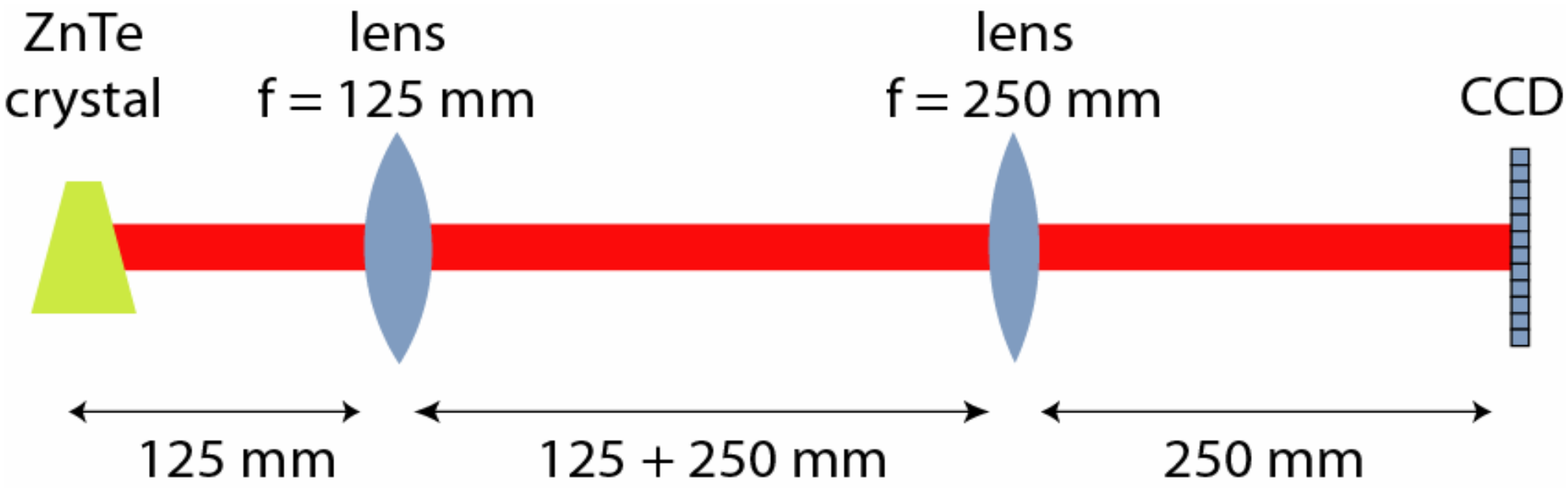
encircled energy

50% of energy within 9 µm diameter



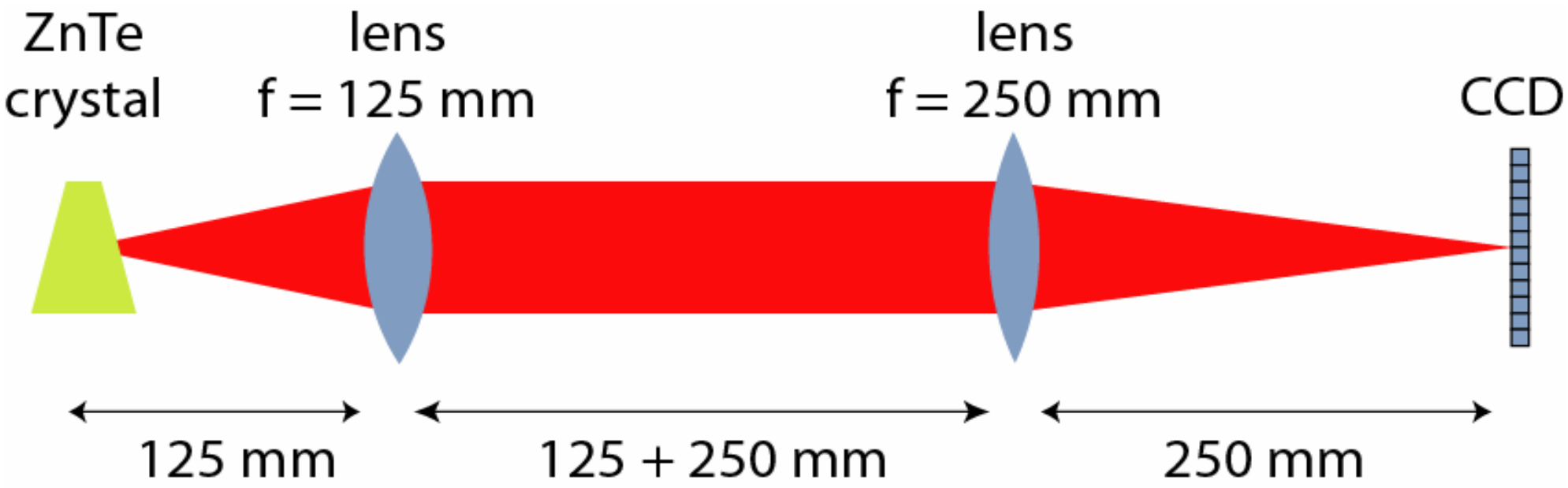
Optics for the TEO experiment

Optics (simplified)



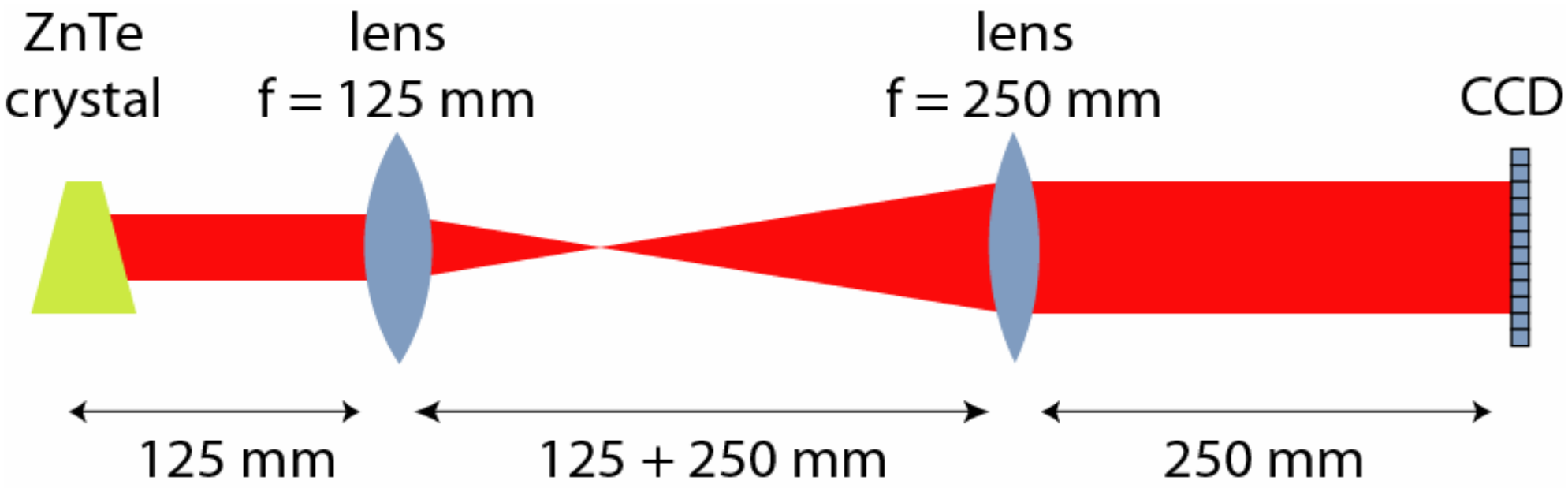
Optics for the TEO experiment

Imaging of a Point on the Crystal



Optics for the TEO experiment

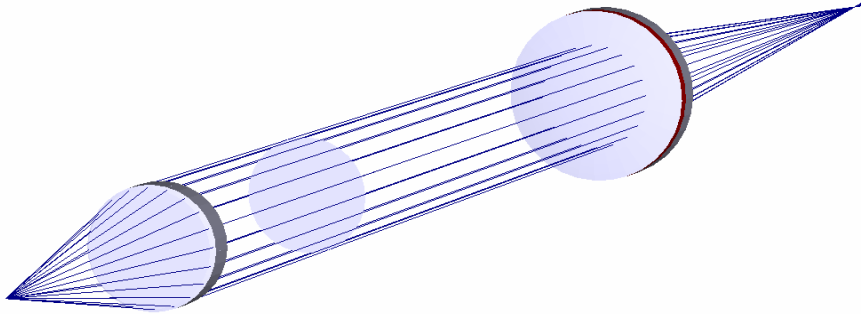
Envelope of the Laser



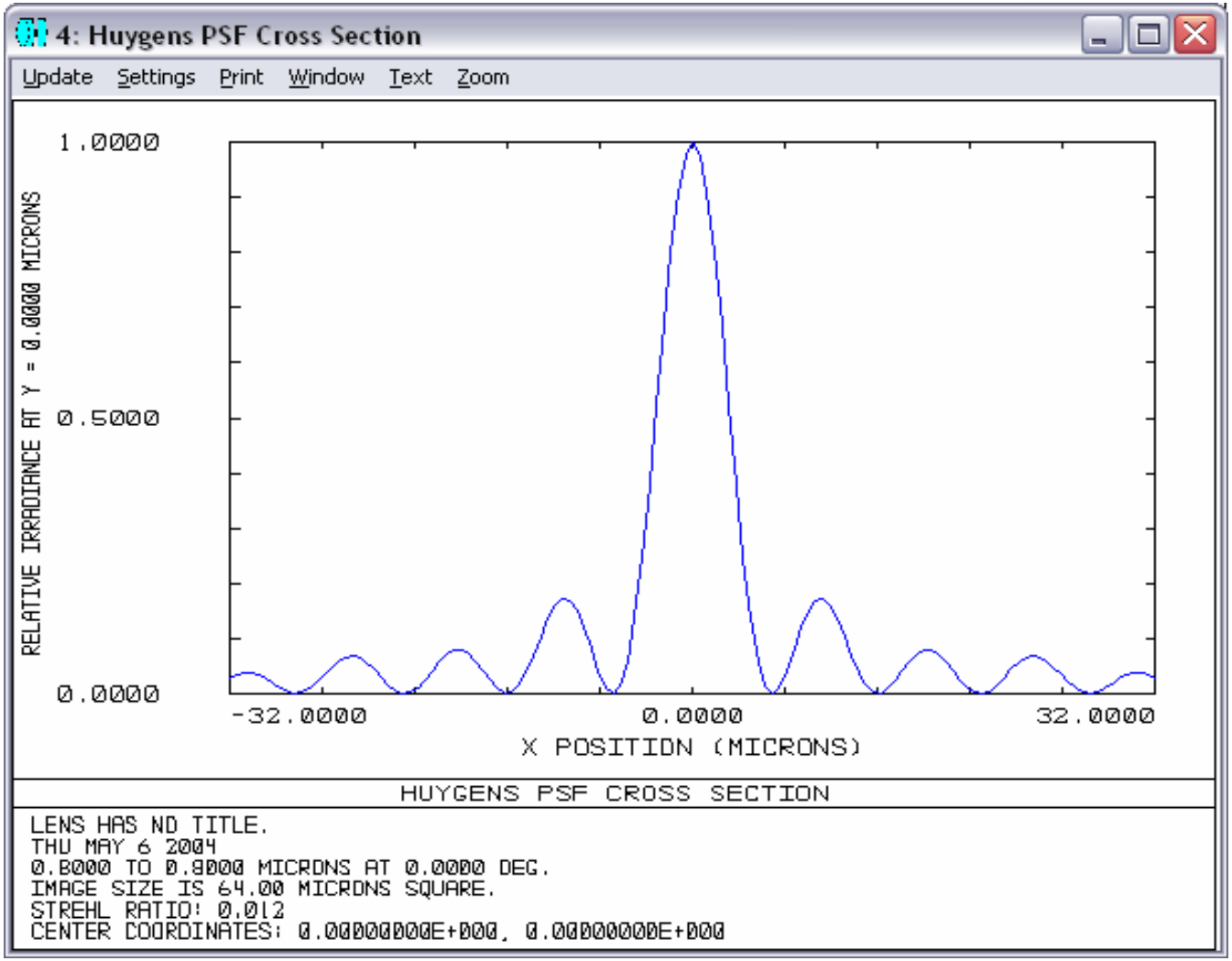
⇝ beam expander optics

Optics for the TEO experiment

ZEMAX simulations



- Point Spread Function



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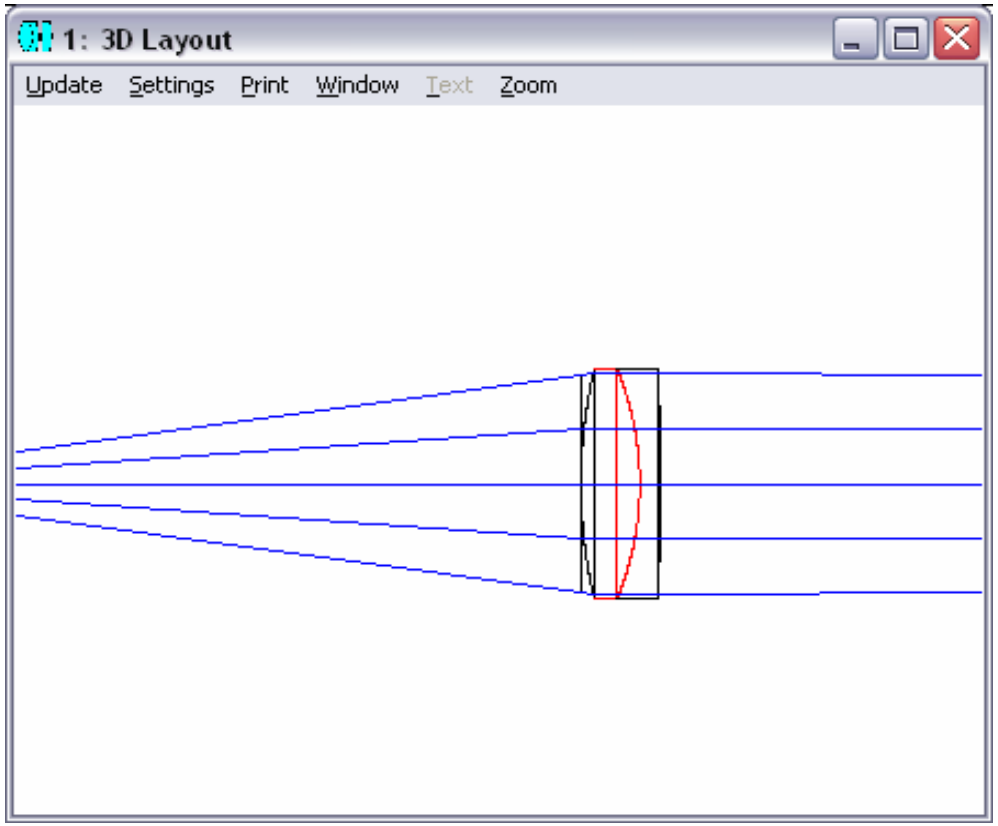
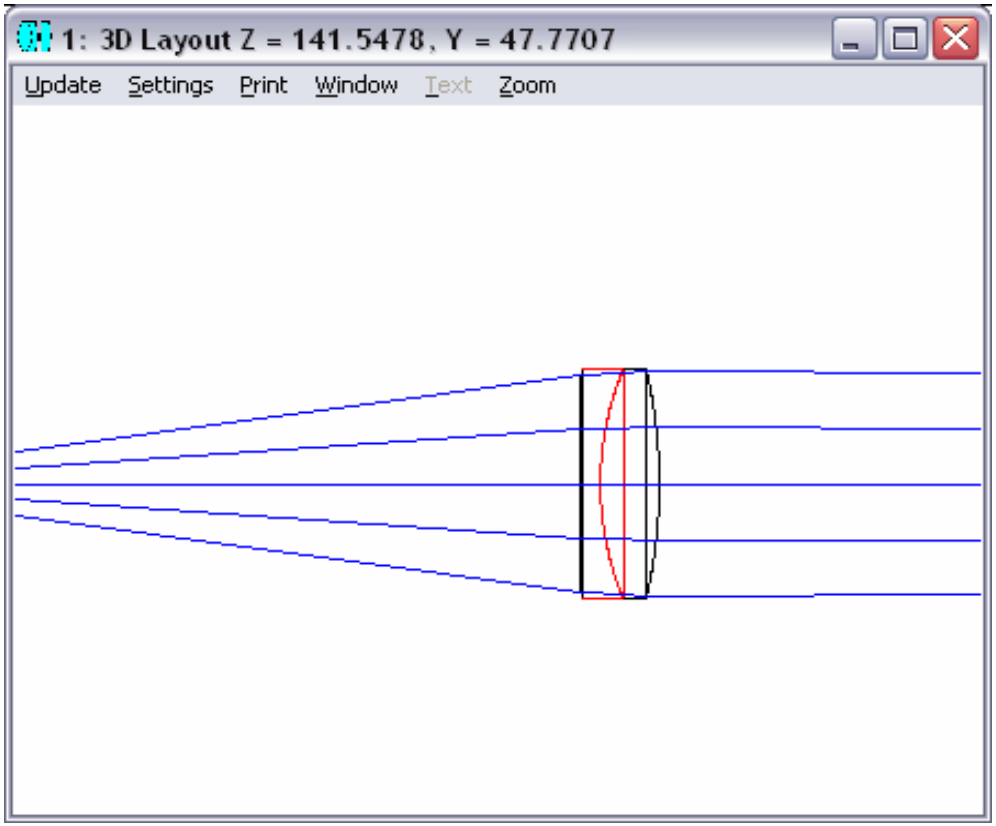
Optics for the TEO experiment

Reverse the first doublet

- Distribute the refraction evenly over all surfaces

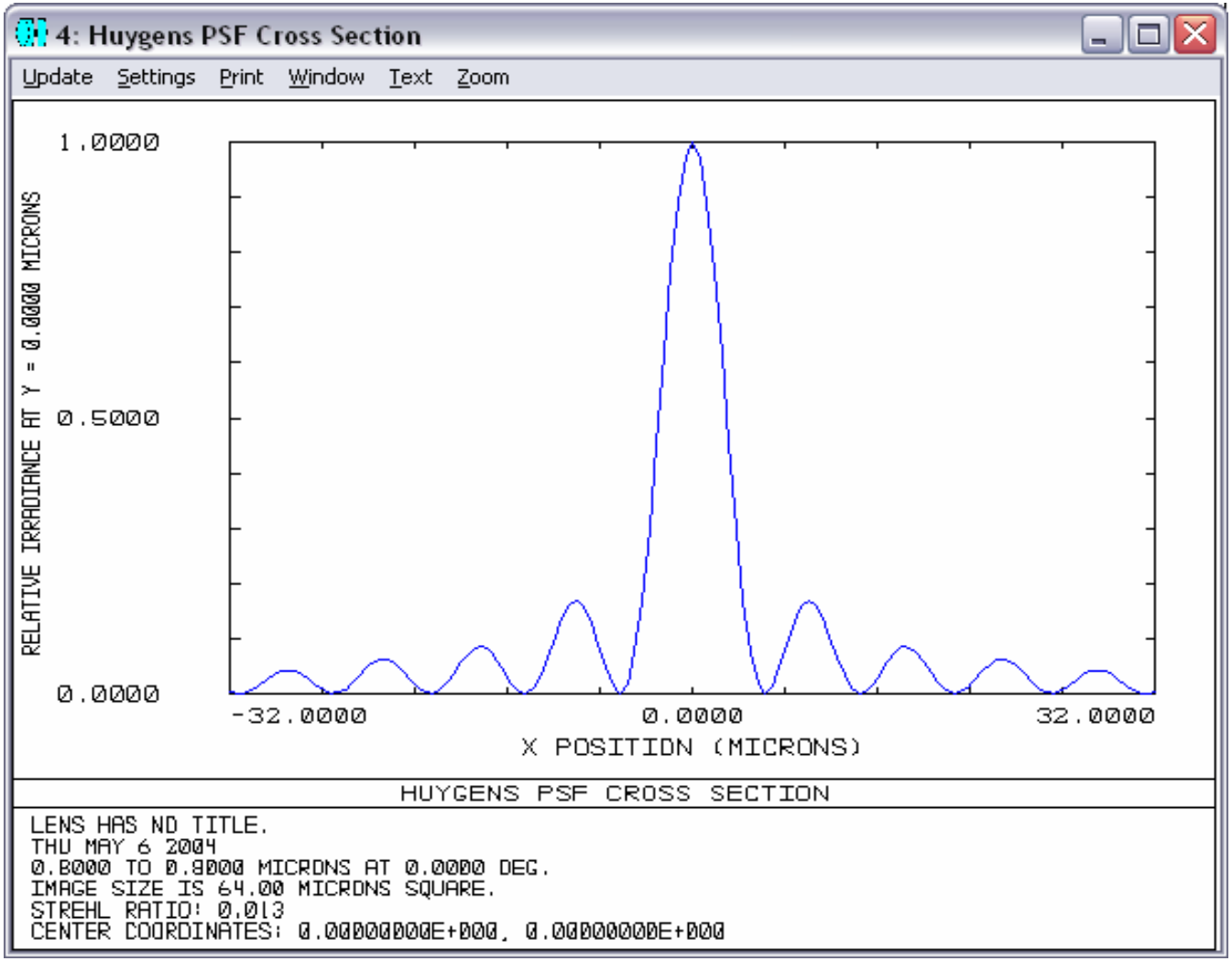




Optics for the TEO experiment

Improved resolution

- The reversal of the first doublet improves the resolution:



Optics for the TEO experiment

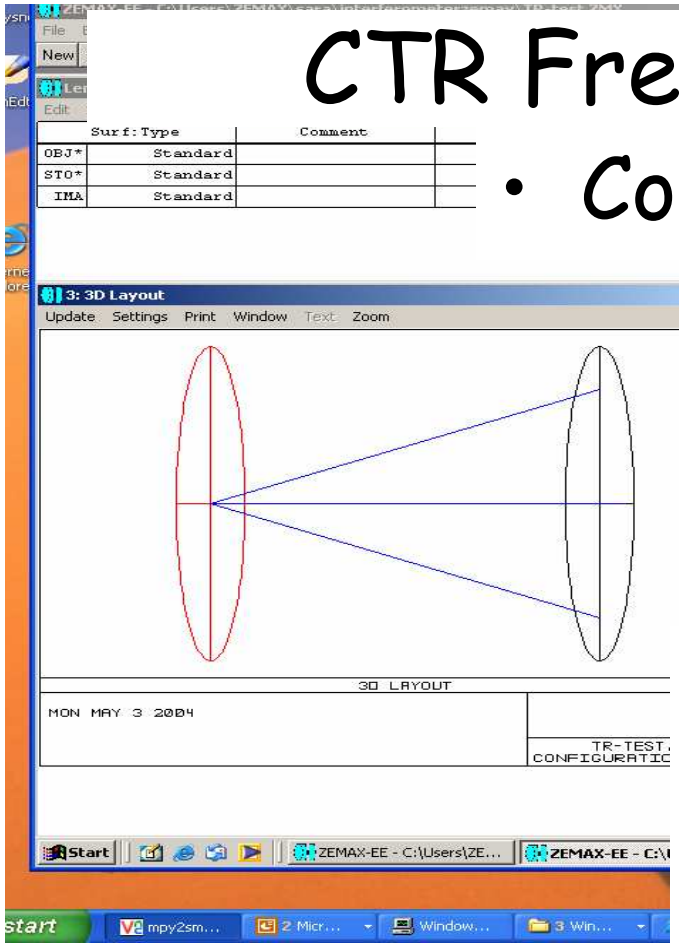
ZEMAX Simulations

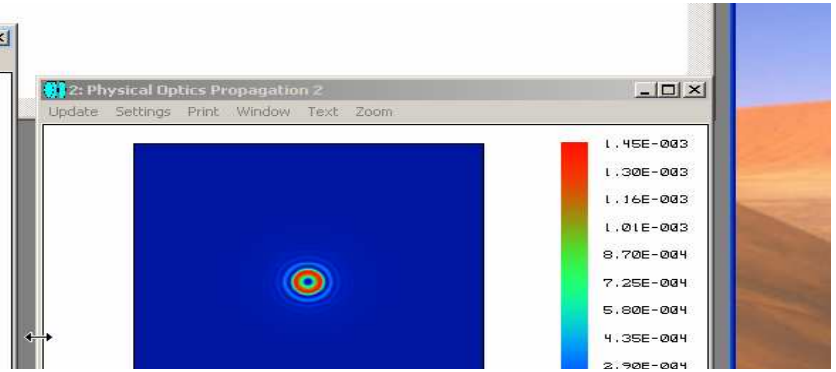
Further studies (Stefan Düsterer)

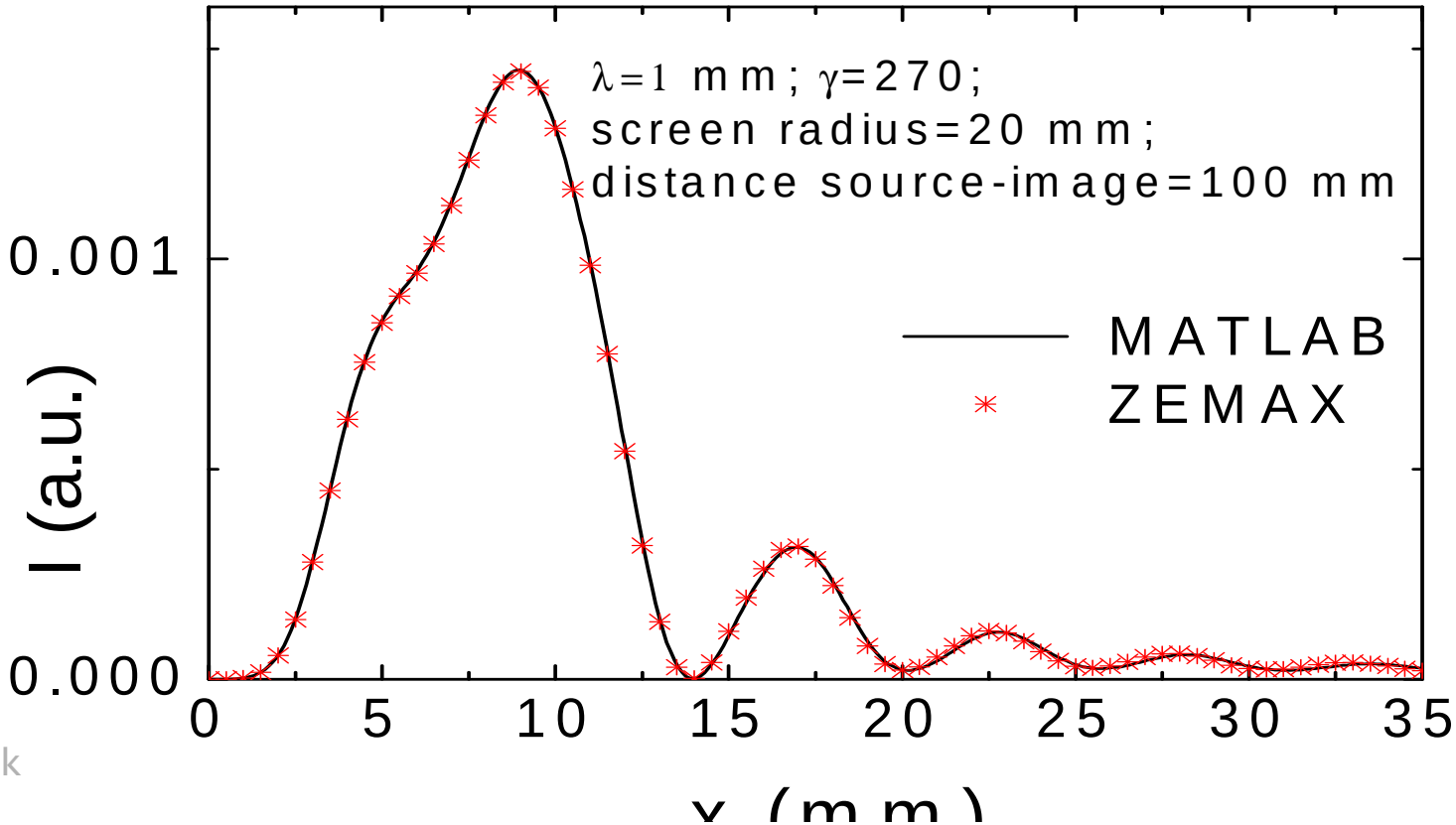
- Optimization of the distances
- Optimization of the tilt of the camera
- Calculation of the PSF with Fresnel diffraction
- Limitation of the resolution: spherical aberrations
  - ⇒ It does not help to increase the diameter of the lens
- Calculation of the resolution for off-axis points
- Studies on the required alignment precision

CTR Free Propagation in Near Field

- Comparison with MATLAB code



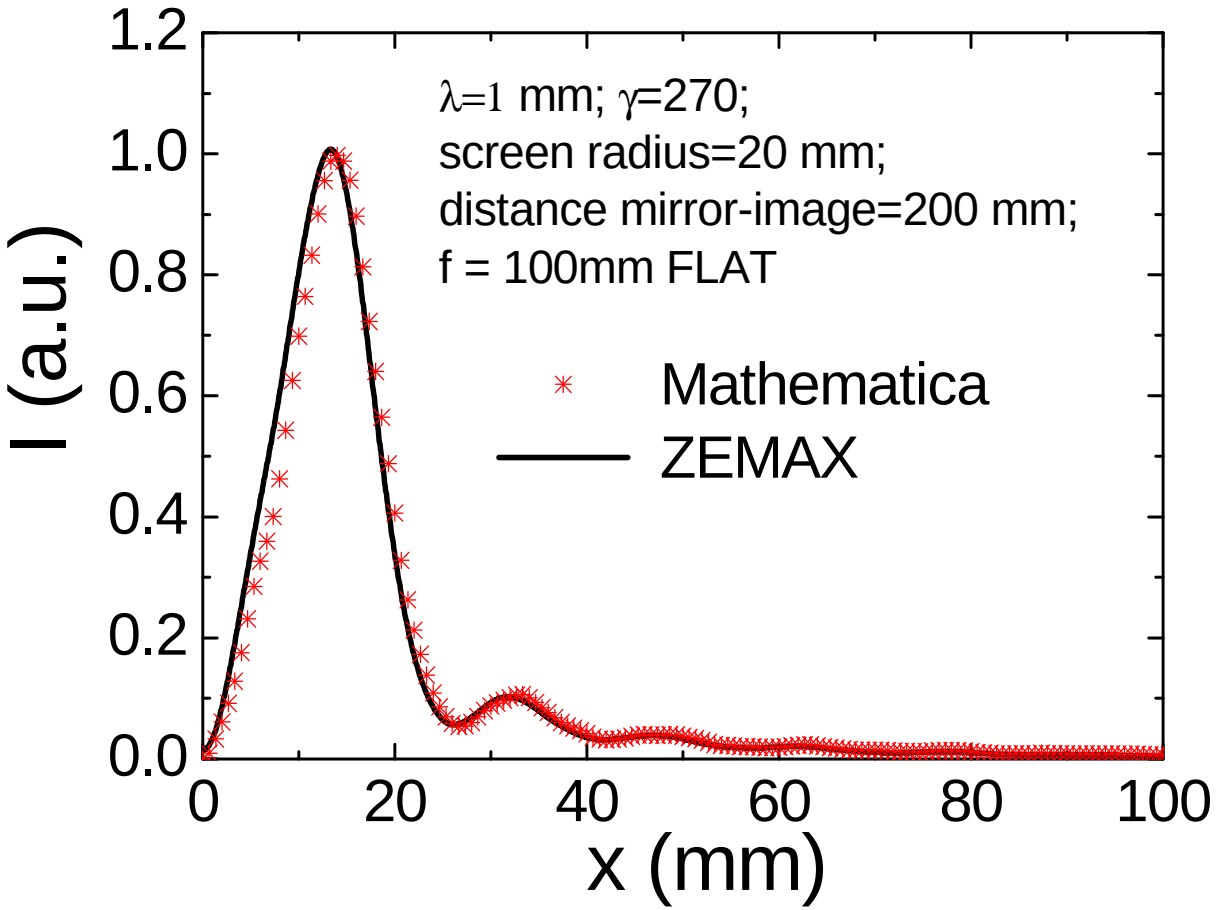


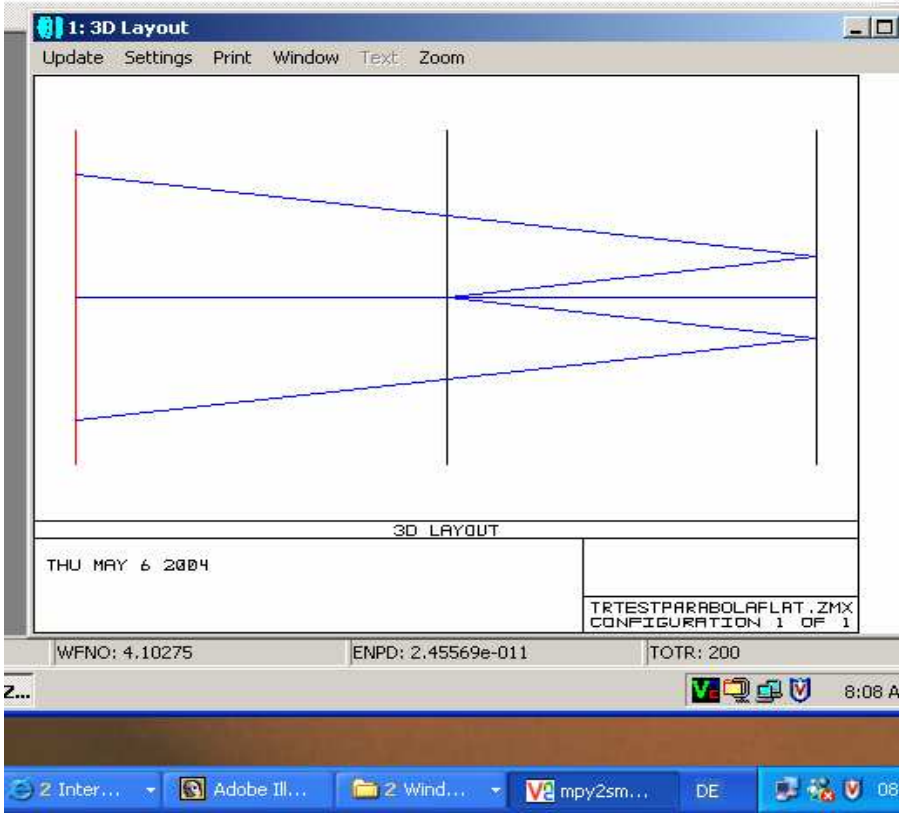


CTR on flat mirror

Comparison with Bernhard's code

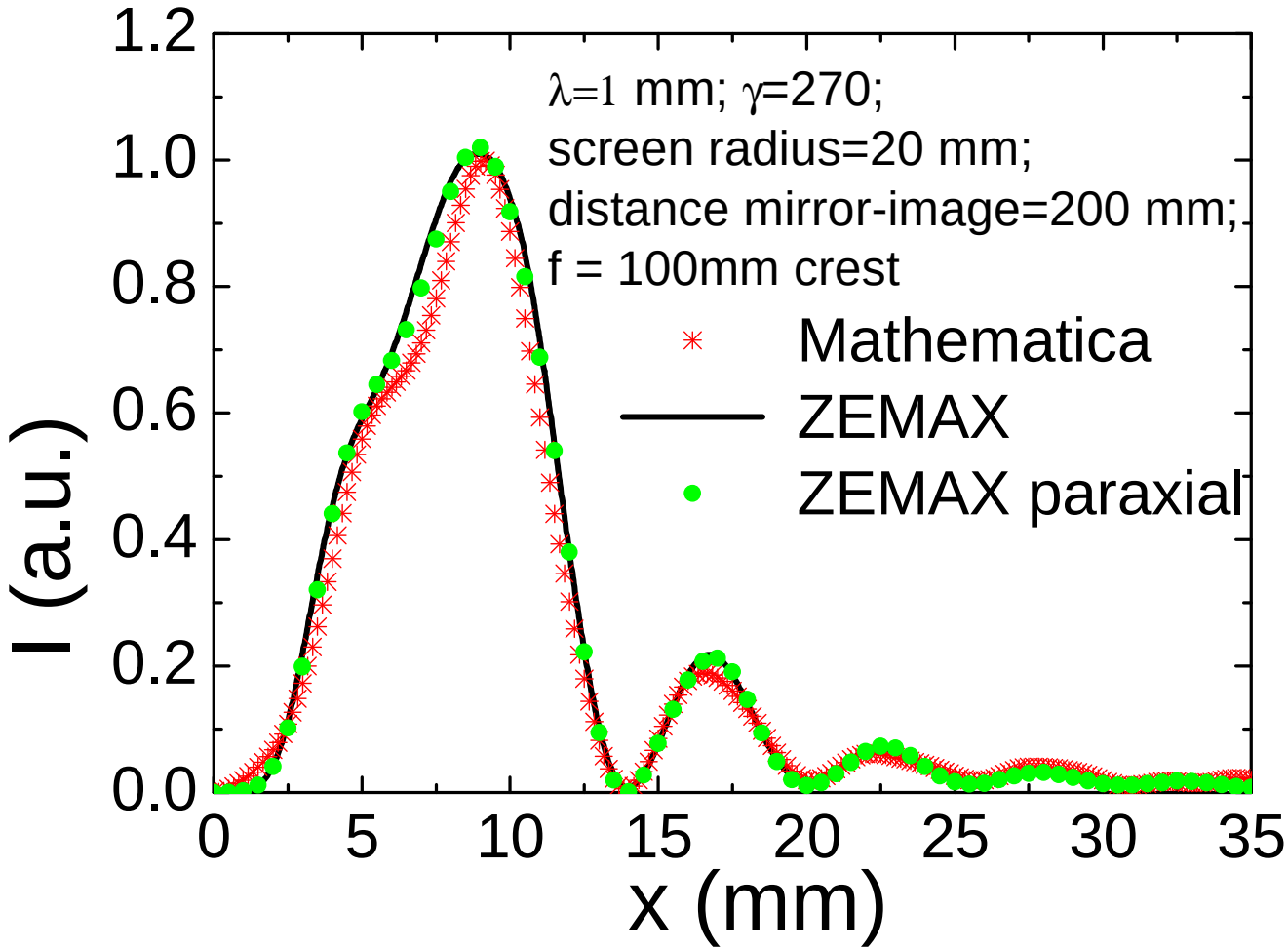
OBJ	Standard		Infinity	1.000000E-010			0.000000		0.000000
STO*	Standard		Infinity	100.000000			50.000000	U	0.000000
2*	Standard		Infinity	-200.000000		MIRROR	100.000000	U	0.000000
IMA	Standard		Infinity	-			50.000000	U	0.000000

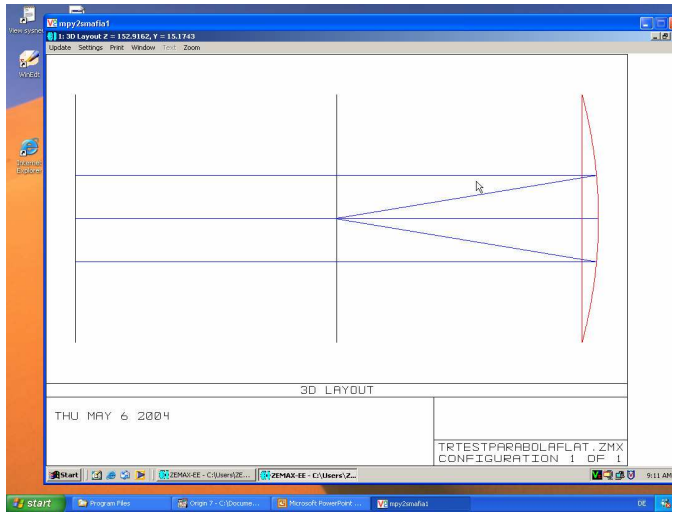


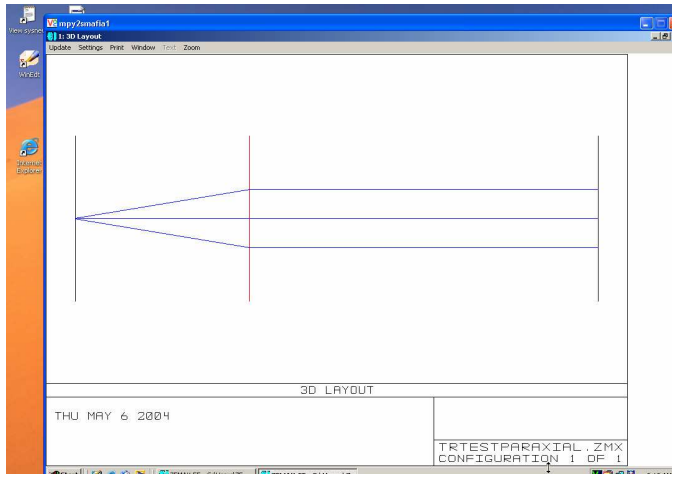


CTR on paraboloid mirror: crest

Comparison with Bernhard's code

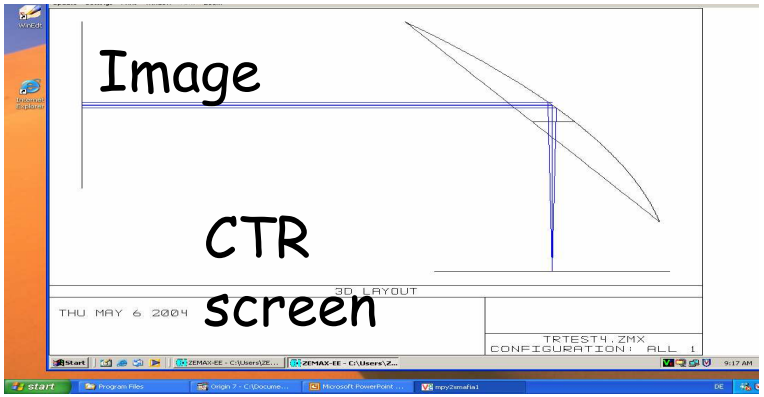




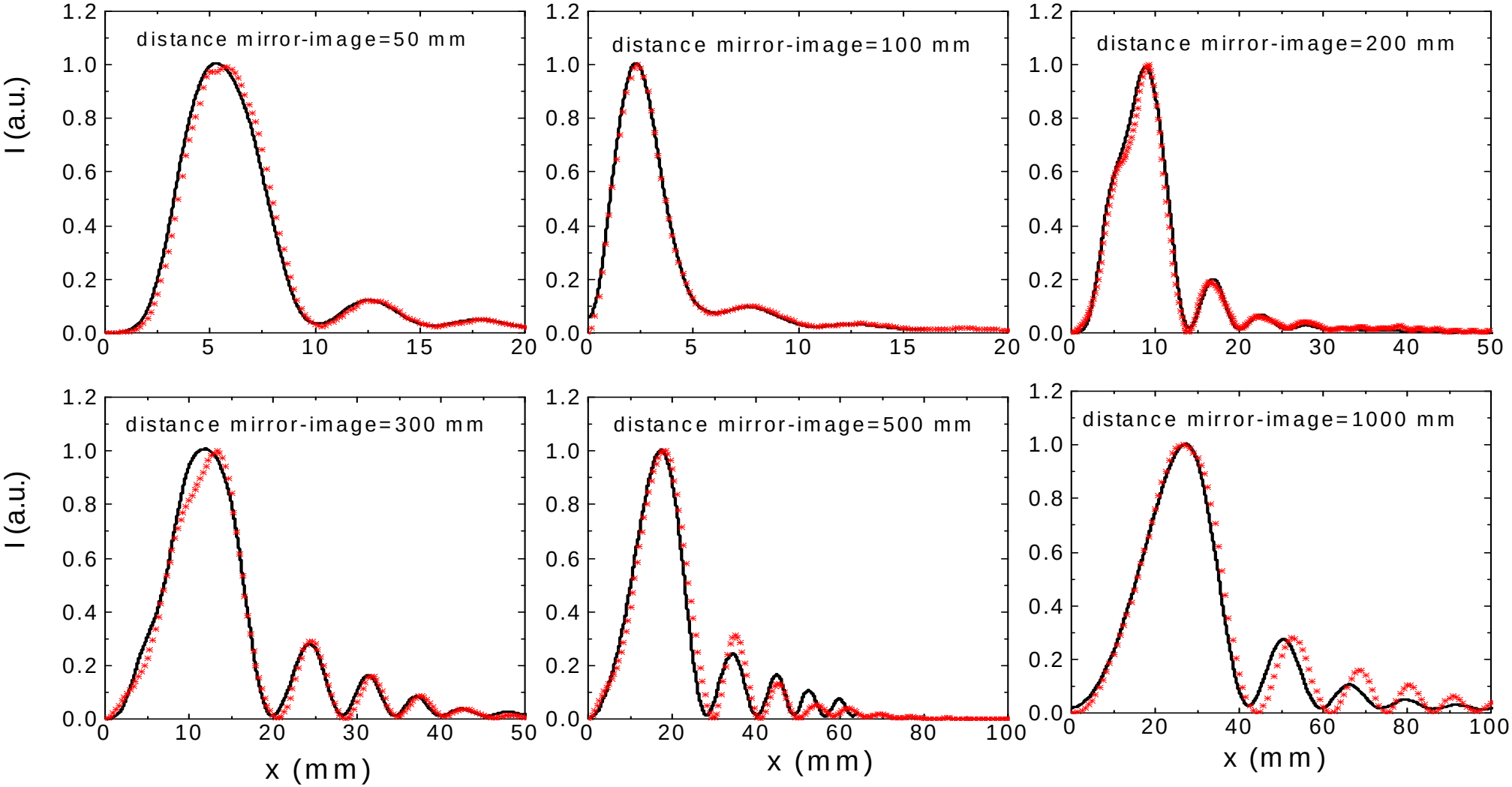


CTR on paraboloid mirror

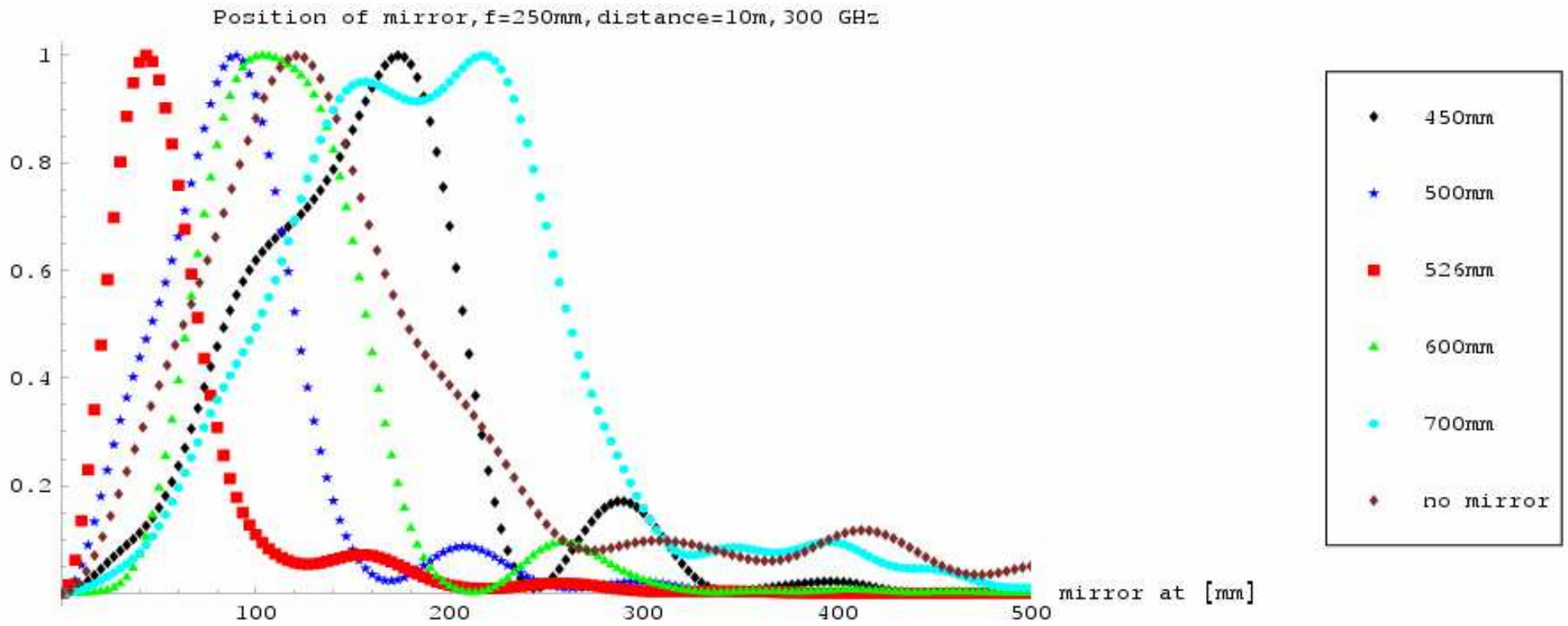
Comparison with Bernhard's code



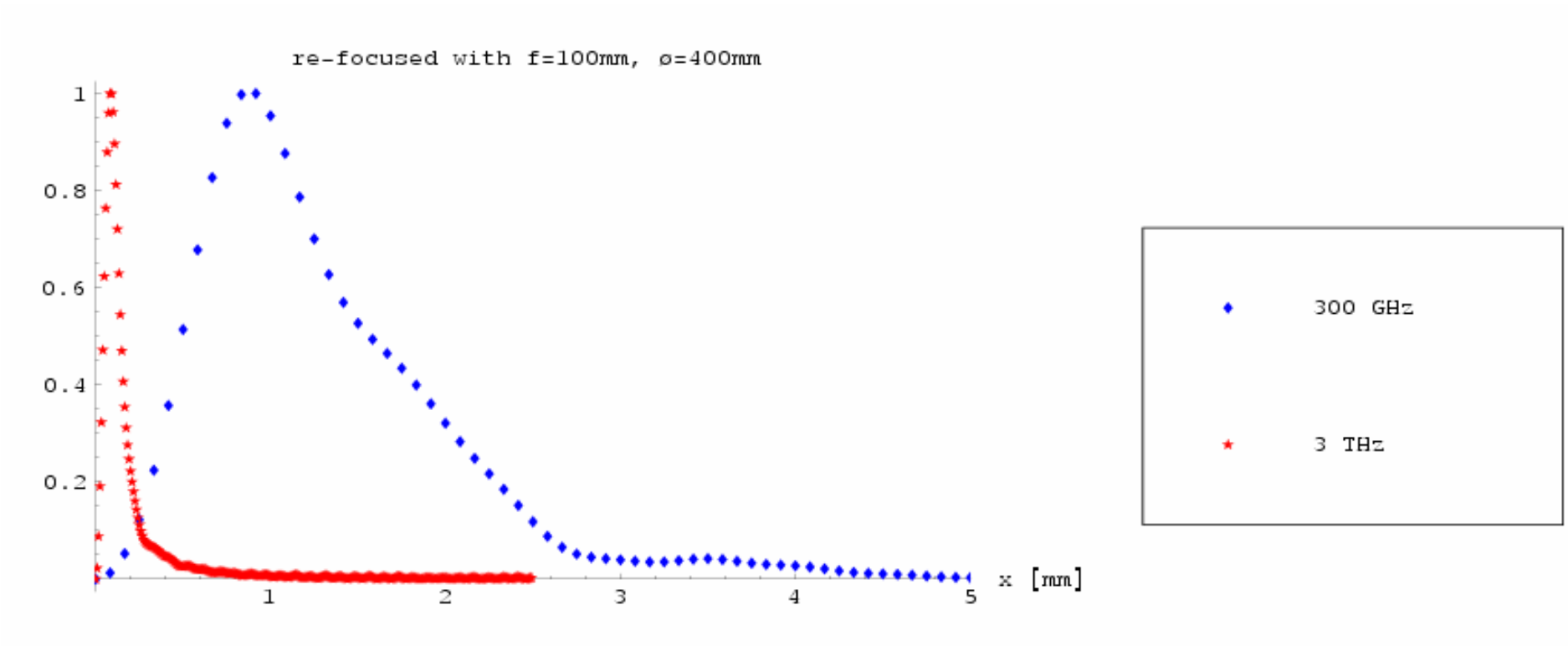
λ=1mm; Y=270; screen radius=20 mm; f = 50mm



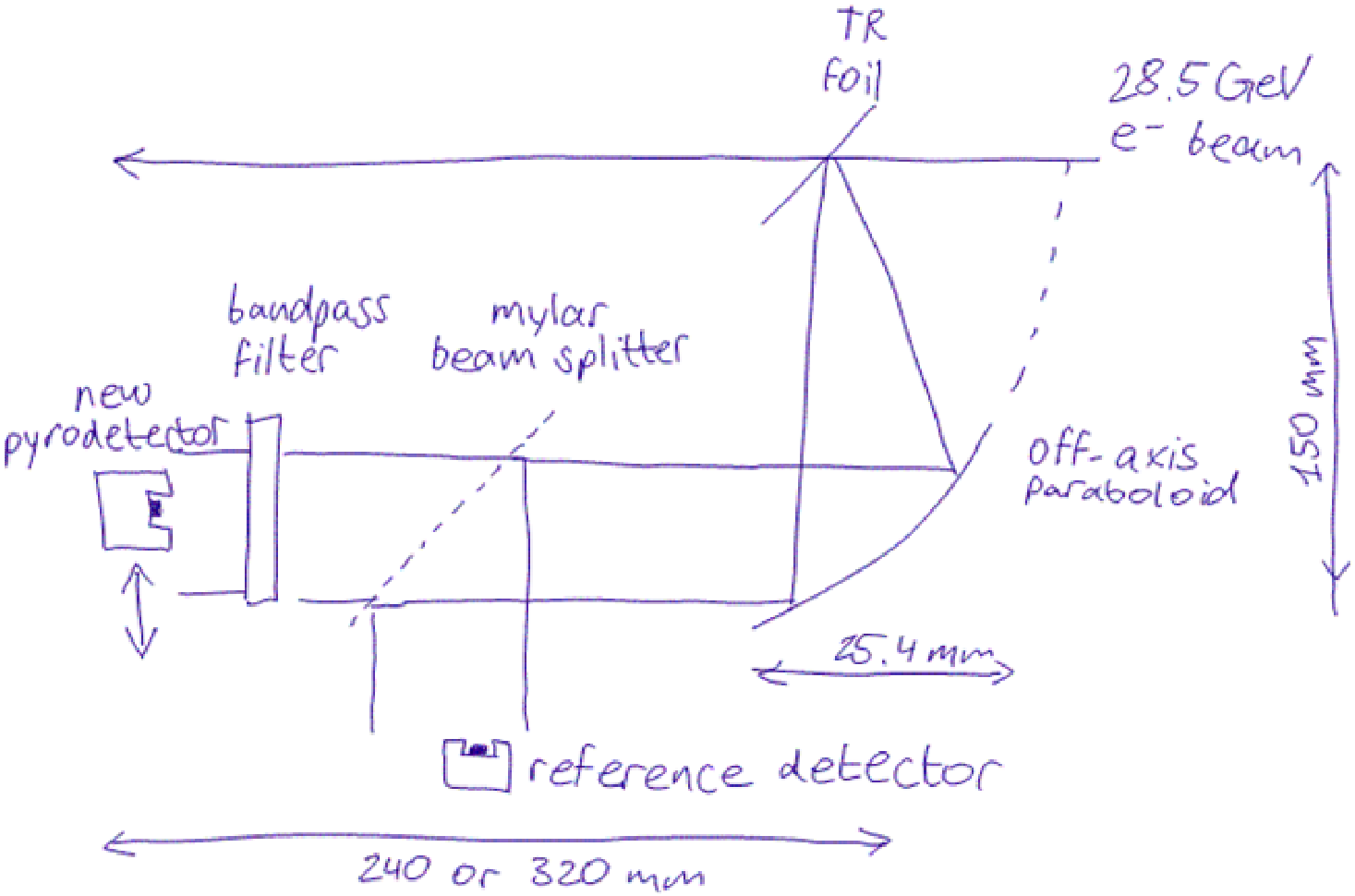
Transmission through a 10m beam line



Re-focusing of the wave

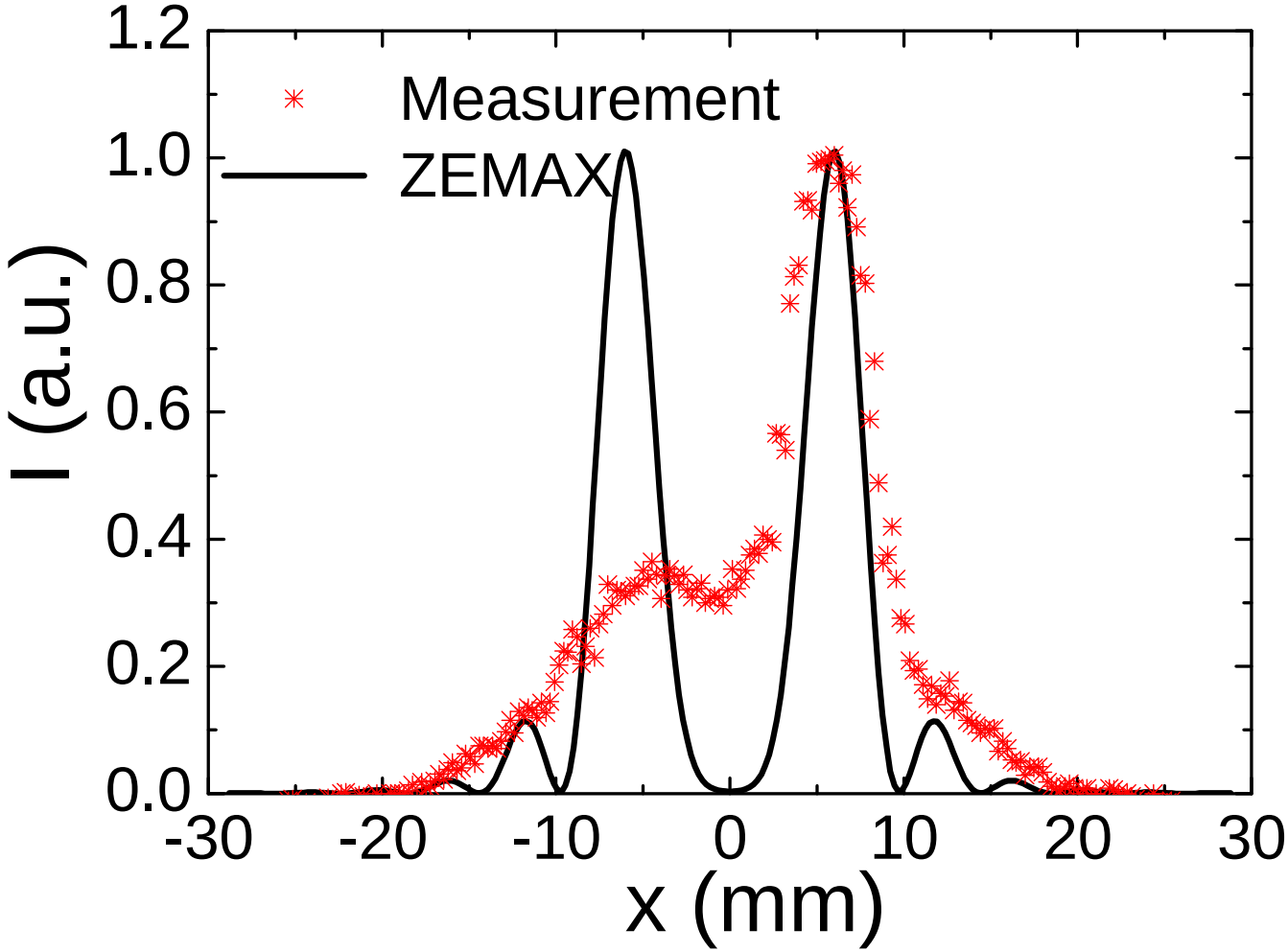


CTR on paraboloid mirror
Experimental setup at SPPS



CTR on paraboloid mirror
Comparison with measurements

λ=0.3 mm; γ=55772; screen radius=20 mm; distance mirror-image=320 mm; f = 75mm



Future plans

- Check the resolution of the TEO optics experimentally
- Check the transfer function for the interferometer in TTF1 (CTR)
- BC3 Frascati transfer line (CDR)
- Implement Surface.DLL (with J. Menzel) for elliptical and paraboloid mirrors for design optimization (at the moment impossible)
- Implement CSR source from O. Grimm calculations
- Check O. Grimm (CSR) transfer line
- Comparison with JLab (CSR) transfer line
- Transfer line for the interferometer @ 140m

Jlab transfer line

mpy2smafia1

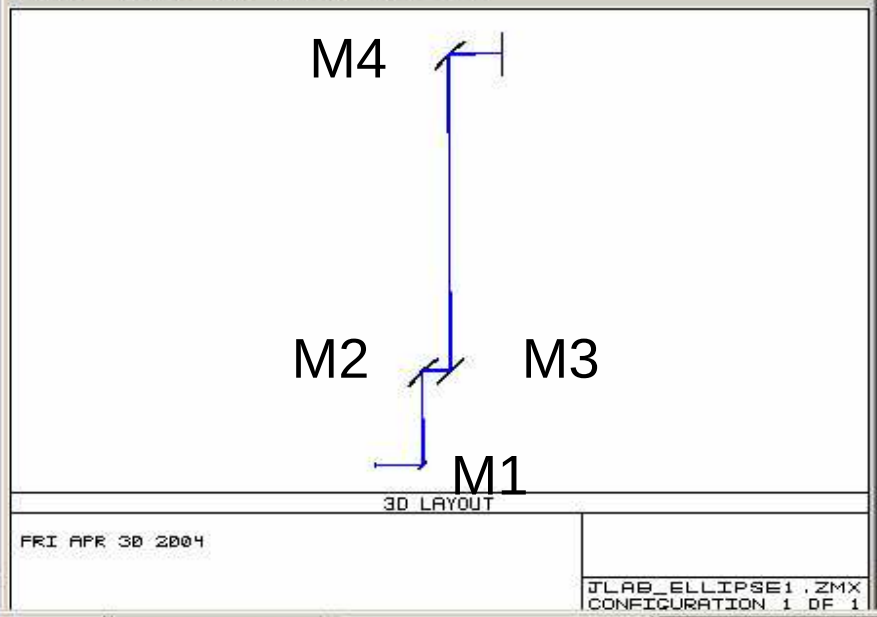
Lens Data Editor

Edit Solves Options Help

	Surf:Type	Comment	Radius	Thickness	Glass		Semi-Diameter		Conic
OBJ	Standard		Infinity	1.000000E-004			0.000000		0.000000
STO*	Standard		Infinity	625.000000			200.000000	U	0.000000
2	Coordinate B..			0.000000	-		0.000000		
3*	Standard	FIRST ELLIPSOID M1	-883.883600	0.000000	MIRROR		50.000000	U	0.500000
4	Coordinate B..			-625.000000	-		0.000000		
5	Coordinate B..			-302.319000	-		0.000000		
6*	Standard	SECOND ELLIPSOID M2	546.256800	302.319000	MIRROR		50.000000	U	-0.651100
7	Coordinate B..			372.000000	-		0.000000		
8	Coordinate B..			0.000000	-		0.000000		
9*	Standard	FLAT MIRROR M3	Infinity	-1.000000E-004	MIRROR		250.000000	U	0.000000
10	Coordinate B..			-4480.000000	-		0.000000		
11	Coordinate B..			0.000000	-		0.000000		
12	Coordinate B..			-302.000000	-		0.000000		
13*	Standard	THIRD ELLIPSOID M4	546.000000	302.000000	MIRROR		50.000000	U	-0.651100
14	Coordinate B..			0.000000	-		0.000000		
IMA	Standard		Infinity	-			300.000000	U	0.000000

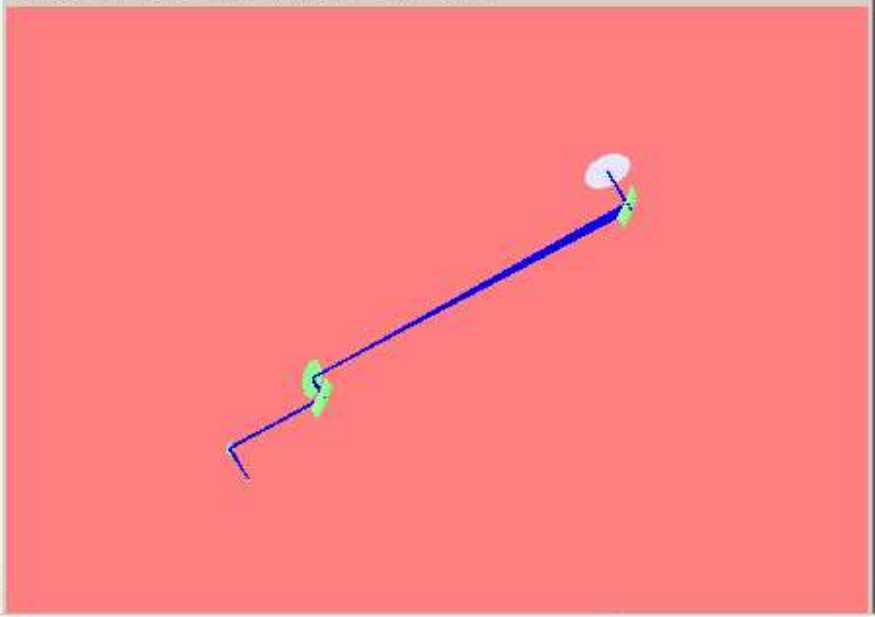
1: 3D Layout

Update Settings Print Window Text Zoom



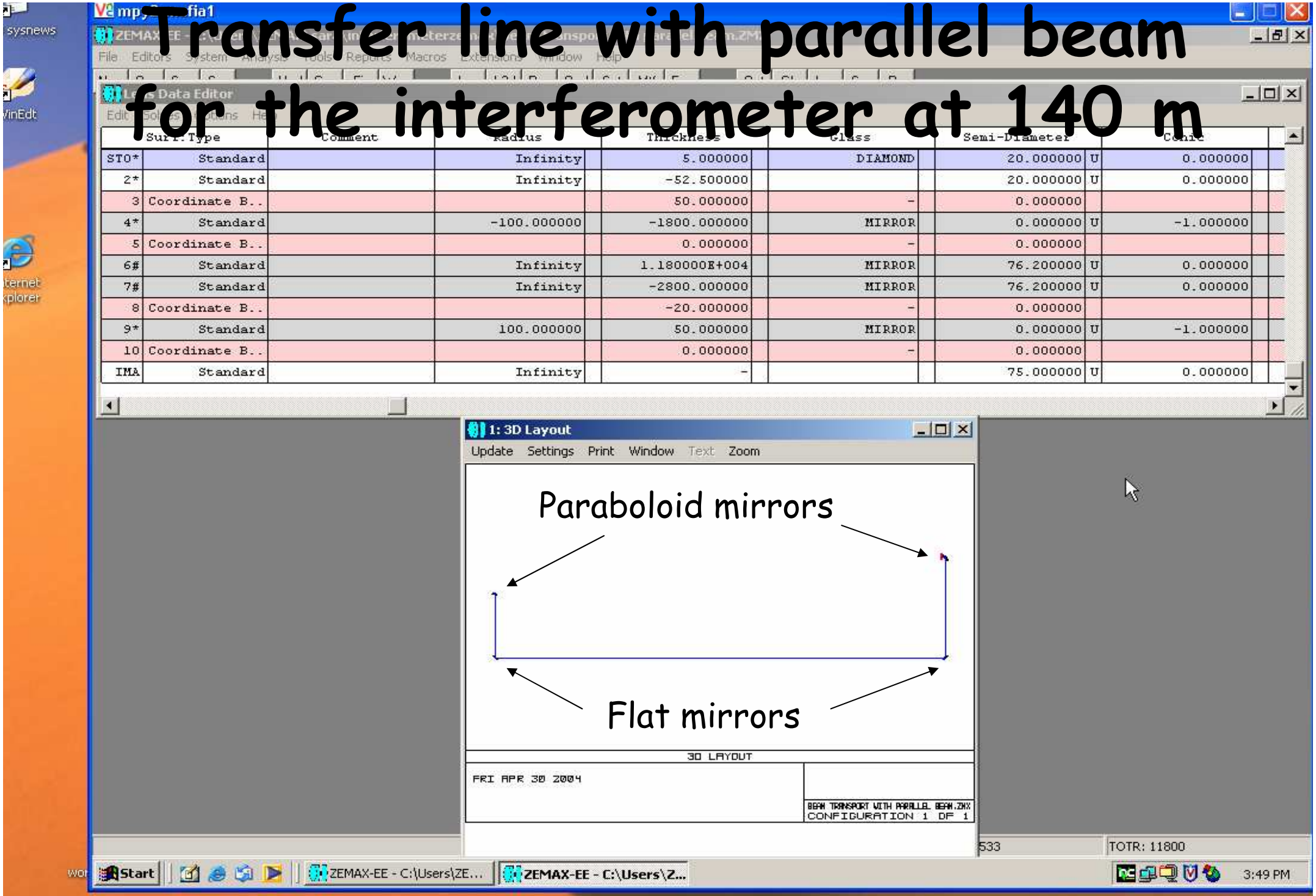
2: Shaded Model

Update Settings Print Window Text Zoom



Start ZEMAX-EE - C:\Users\ZE... ZEMAX-EE - C:\Users\Z... 3:59 PM

Transfer line with parallel beam for the interferometer at 140 m



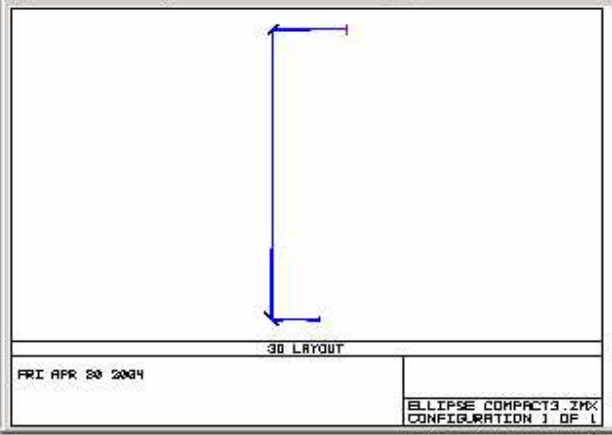
Transfer line with elliptical mirrors for the interferometer at 140 m

ZEMAX-EE - C:\Users\ZEMAX\sara\Ellipticalmirrors\ellipse compact3.zmx

	Surf:Type	Comment	Radius	Thickness	Glass	Semi-Diameter		Conic
1*	Standard	VACUUM WINDOW	Infinity	5.000000	DIAMOND	10.000000	U	0.000000
STO*	Standard		Infinity	-52.095000		10.000000	U	0.000000
3	Coordinate B..	FIRST ELLIPSOID		0.000000	-	0.000000		
4	Standard	FIRST ELLIPSOID	Infinity	47.230700		0.000000	U	0.000000
5*	Standard	FIRST ELLIPSOID	-90.000000	-47.230700	MIRROR	212.000000	U	-0.820000
6	Coordinate B..	FIRST ELLIPSOID		-900.000000	-	0.000000		
7	Standard		Infinity	0.000000		0.000000	U	0.000000
8	Coordinate B..	SECOND ELLIPSOID		0.000000	-	0.000000		
9	Standard	SECOND ELLIPSOID	Infinity	-415.875300		0.000000	U	0.000000
10*	Standard	SECOND ELLIPSOID	780.882400	415.875300	MIRROR	212.000000	U	-0.770300
11	Coordinate B..	SECOND ELLIPSOID		5900.000000	-	0.000000		
12	Standard		Infinity	0.000000		0.000000	U	0.000000
13	Coordinate B..	THIRD ELLIPSOID		0.000000	-	0.000000		
14	Standard	THIRD ELLIPSOID	Infinity	6682.000000		0.000000	U	0.000000
15*	Standard	THIRD ELLIPSOID	-1132.000000	-618.000000	MIRROR	212.000000	U	-0.690000
16	Coordinate B..	THIRD ELLIPSOID		1400.000000	-	0.000000		
IMA	Standard		Infinity	-		200.000000	U	0.000000

1: 3D Layout

Update Settings Print Window Text Zoom



EFFL: 0.902574 | WFNO: 28.3905 | ENPD: 20 | TOTR: 13779.9